



University of Moratuwa

Wisdom is all wealth

Noise and Interference in Instrumentation

Part 1: Introduction

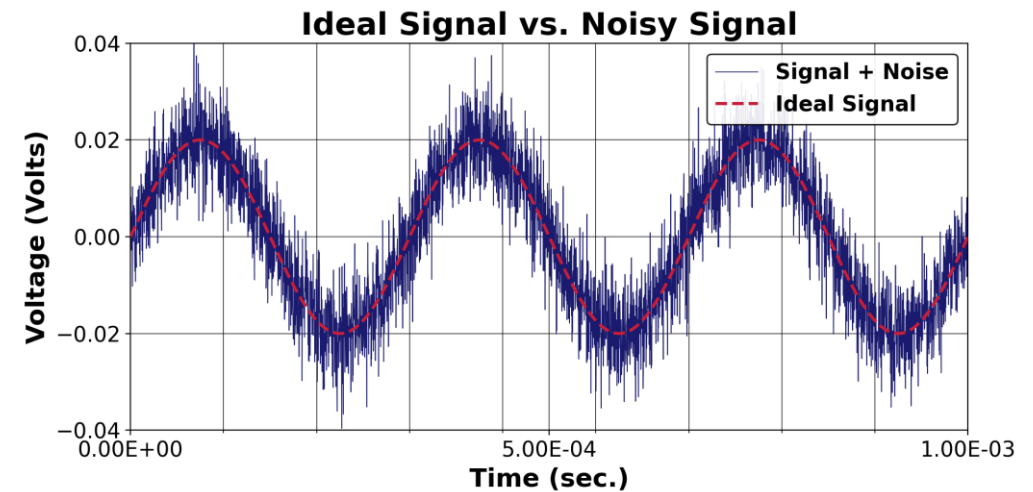
Anton F. P. Van Putte, Electronic measurement systems, Taylor & Francis, 2nd Edition

M. M. S. Anand, Electronic instruments and Instrumentation Technology, PHI

Noise

Introduction and Motivation

- Any unwanted or random fluctuation that interferes with a signal of interest can be considered as noise.
- Noise is everywhere and it
 - Contaminates signals,
 - Degrades accuracy, precision, resolution,
 - Reduces device reliability.
- Understanding noise characteristics helps:
 - Design optimization – Informs source-load matching, component choice, filtering/bandwidth, etc.
 - Provides an uncertainty floor – Noise sets the minimum reliable limit of detection for measurements.
 - Compliance – Regulations require that devices meet certain noise standards.



Noise

Types of Noise

- Inherent Transmitted Noise:
 - This is the noise in the physical quantity that a sensor measures – Difficult to characterize unless the source is known.
 - It could also be noise that is introduced by electronic stages prior to the sensor.
- Intrinsic Noise
 - This is the noise that originates from within a device or a system.
 - All practical instruments generate noise, and when a signal passes through such an instrument, noise contaminates and degrades the signal.
 - Ex: Thermal noise in a resistor, shot noise in a diode, etc.
- Interference Noise:
 - Generally considered separate from intrinsic noise.
 - Originates from external noise sources that couple into the system, hence not an intrinsic quantity of a system.
 - However, the **amount** of interference noise **coupled** could be an intrinsic quantity dependent on system design.
 - Ex: Powerline interference, Interference from solar flares, lightning, etc.

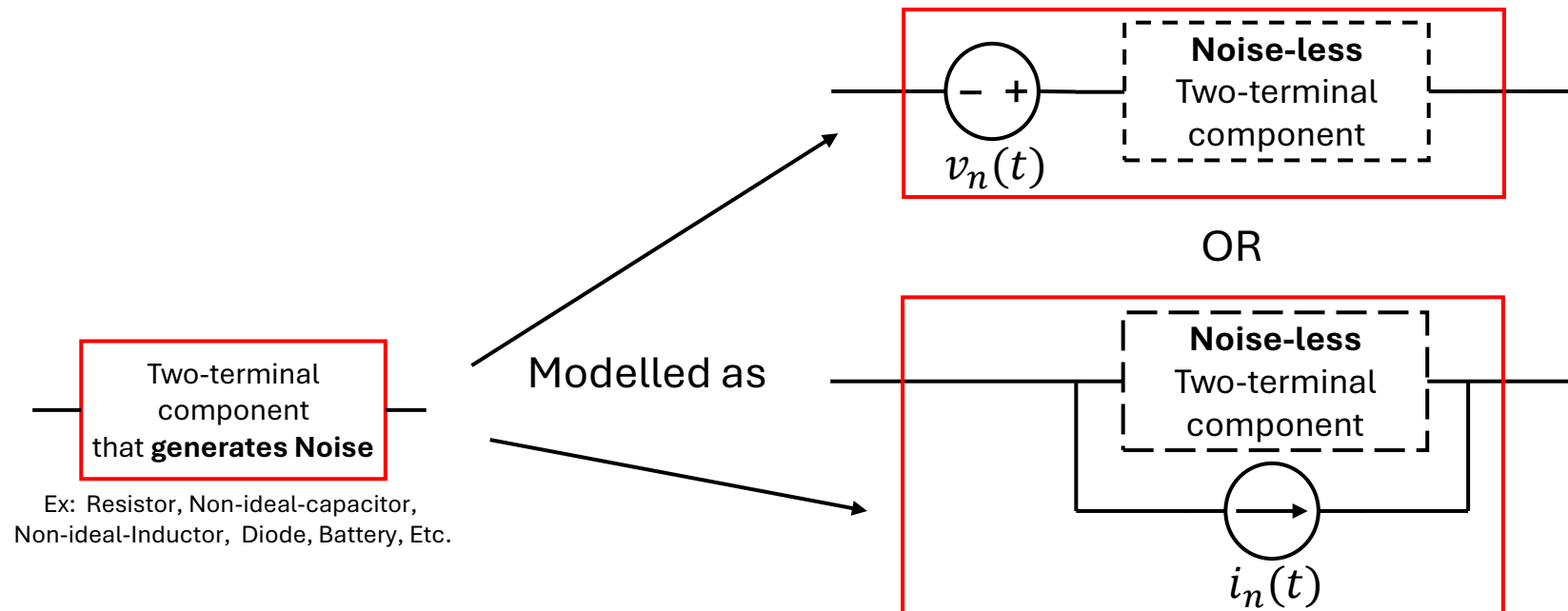
Noise

Modelling Noise

- Modelling noise
 - Enables the simplification of calculations,
 - Provides insights into noise mitigation strategies,
 - Allows better design optimization.

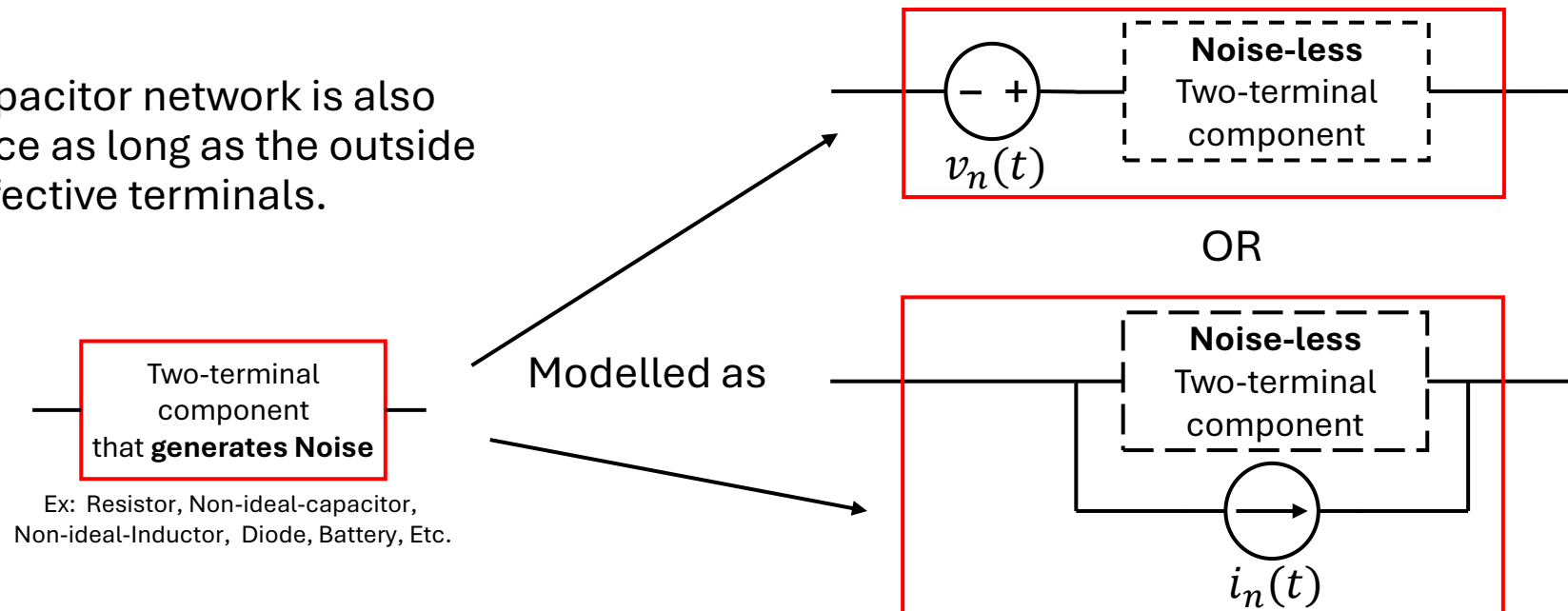
Modelling noise: 1-Port Devices

- For a simple **two-terminal (1-port)** device:
 - Modelled with a **single** series voltage source **OR** parallel current source applied on an ideal noiseless version of the device.



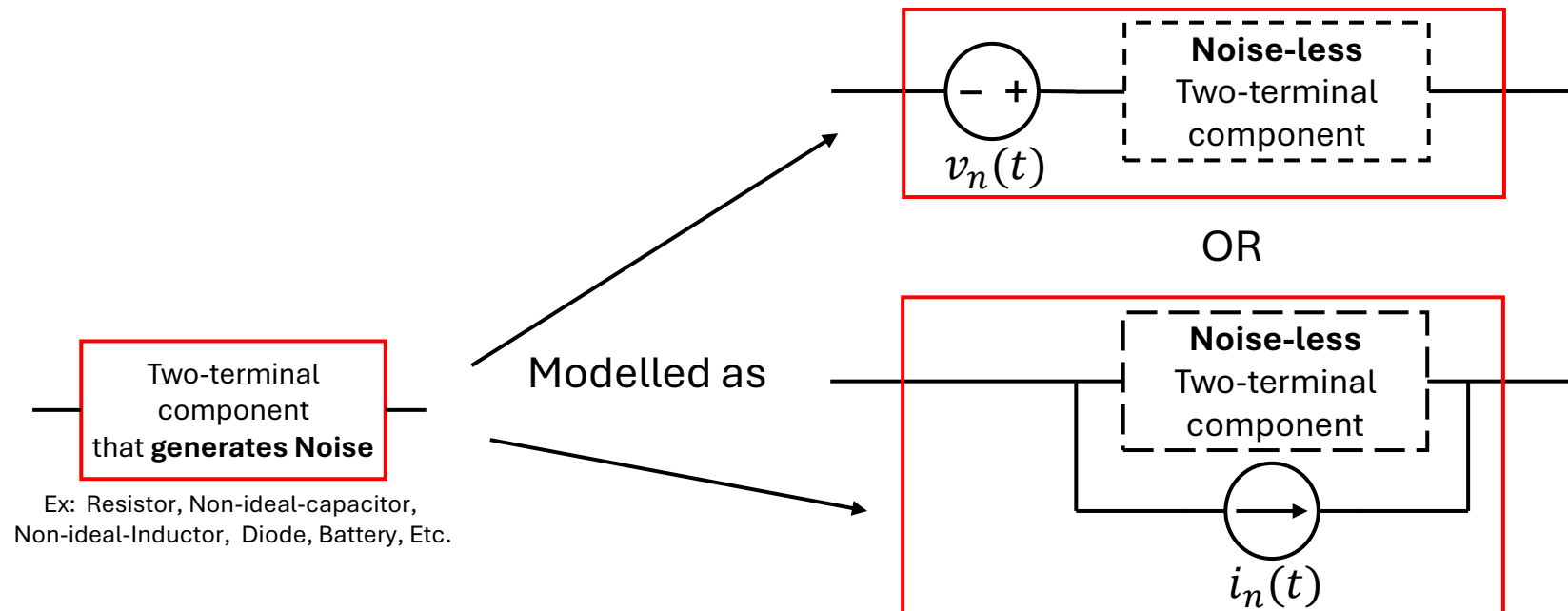
Modelling noise: 1-Port Devices

- For a simple **two-terminal (1-port)** device:
 - Modelled with a **single** series voltage source **OR** parallel current source applied on an ideal noiseless version of the device.
 - You can go between the voltage source (Thevenin's equivalent circuit form) and the current source (Norton's equivalent circuit form).
 - Note: This only works for systems that are linear or at least can be characterized as linear around an operating point.
 - Note: A resistor network or a capacitor network is also considered a simple 1 port device as long as the outside world only interacts with two effective terminals.



Modelling noise: 1-Port Devices

- For a simple **two-terminal (1-port)** device:
 - However, depending on the type of device, and the type of noise it generates, using one could be preferred over the other.
 - Ex:
 - For a resistor, either is fine.
 - For a diode, the current source would be better (discussed later).



Modelling noise: 2-Port Devices

- For a **3-terminal** or **4-terminal (2-port)** device:
 - Ex: Transistor, op-amp, etc.
 - Cannot model all noise sources with a **single** noise voltage or noise current.

Modelling noise: 2-Port Devices

- For a **3-terminal** or **4-terminal (2-port)** device:
 - Ex: Transistor, op-amp, etc.
 - Cannot model all noise sources with a **single** noise voltage or noise current.
 - We need to use multiple noise voltage/current sources placed at the right places.
 - This is too complex and device dependent:
 - Ex: A BJT has noise sources in its base, base–emitter junction, collector-emitter junction, and a different active device like an opamp will have noise at locations specific to its layout.
 - So we transfer all noise sources to the input side of the device.
 - This is called the **input-referred** noise model.

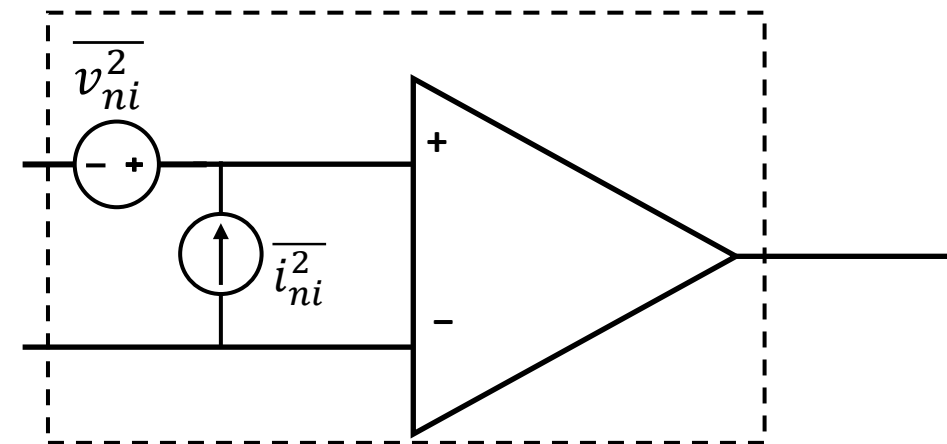
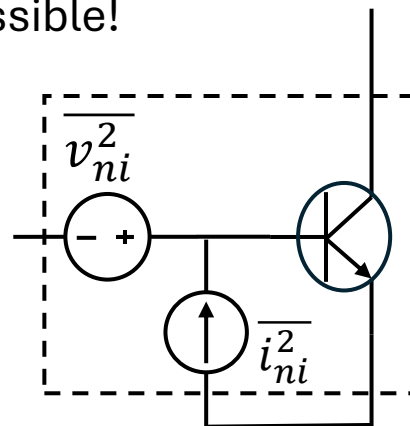
Modelling noise: 2-Port Devices;

Input Referred Noise Model

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 - Ex: A BJT has noise sources in its base, base–emitter junction, collector–emitter junction, and a different active device like an opamp will have noise at locations specific to its layout.
 - So we transfer all noise sources to the input side of the device.
 - This is called the **input-referred** noise model.
 - Input-referred noise model
 - Any complex 2-port device which has noise sources at different locations within it are all collapsed to a **series** voltage source **AND** a **parallel** current source at the **input**.
 - This way, you can do calculations regarding noise agnostic to what is actually inside the two-port device.

Modelling noise: 2-Port Devices; Input Referred Noise Model

- More about input-referred noise model:
 - Even the output side of a device (ex: collector current of a BJT amp) will have its own noise.
 - However, we can do calculations to make it look like the noise actually happened on the input side but got transferred onto the output side via some gain or transconductance.
 - This is how we arrive at the series noise voltage and parallel noise current in the input-referred model.
 - Being able to represent all the noises inside the device as if they were external noises at the input of an otherwise noiseless device allows:
 - Direct comparison of signal magnitude with the calculated noise values referred to the input.
 - A direct calculation of SNR becomes possible!
- More about this later.



Modelling noise:

How to calculate noise voltage/current?

- Now that we know how to model the noise in a circuit via series and parallel noise voltage sources:
 - How do we calculate values for these sources?
 - i.e., $v_n(t)$, $i_n(t)$
 - What do we need to know about the noise to do these calculations?
 - What parameters of noise should we consider?
 - Do instantaneous noise voltage/current values even make sense?
 - No? – They are totally random!
 - Cannot use for any reasonable calculation.
 - At best, we can measure them over a long period of time to arrive at a probability distribution for the occurrence of different noise values.
 - This distribution typically has zero mean.
 - But what about the standard deviation? – This is where the RMS comes in!

Modelling noise:

What parameters to calculate? – 1. RMS

- What is the average noise power?
 - If we know the average noise power, we can calculate the V_{RMS} and the I_{RMS} for any system that has resistance.
 - Even a capacitor has a resistance (non-ideal capacitor).
 - An amplifier can have an equivalent input resistance and output resistance.
 - Etc.
 - But why power, and not just V_{RMS} and I_{RMS} ?
 - Because RMS voltages or currents cannot simply be added together to find the total noise due to a combination of multiple sources.
 - However, power can! Caution: Given that the noise sources are uncorrelated/independent.
 - $P_{Total} = P_1 + P_2 + \dots$
 - An explanation as to why is given in a future slide.
 - But we eventually also talk about V_{RMS} and I_{RMS} because that is what a meter would eventually measure, and that is what the visible effect of noise on a circuit would be.

Modelling noise:

What parameters to calculate? – 2. PSD

- Is this RMS the same for all the different noise frequencies, or does it have a frequency dependent profile?
 - Why are we concerned about frequency?
 - A lot of real-world systems have limited bandwidth.
 - If you take a landline telephone and sing into it, it wouldn't sound as nice at the receiver: Bandwidth limited!
 - If a bat screeched into the microphone, the receiver wouldn't even hear it.
 - So, not all unwanted signals (noise) pass through a given system equally.
 - At the same time, not all sources of noise generate the same frequencies.

Modelling noise:

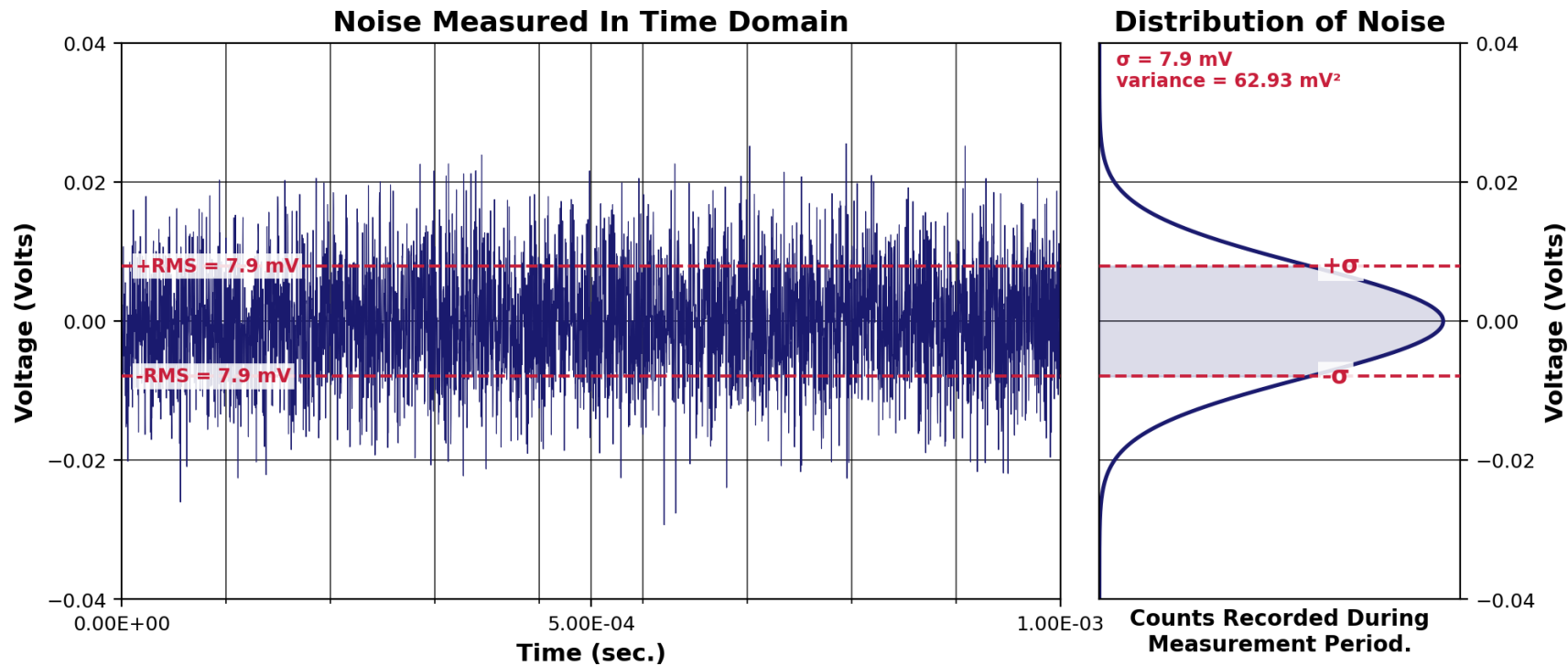
What parameters to calculate? – 2. PSD

- Is this RMS the same for all the different noise frequencies, or does it have a frequency dependent profile?
 - Noise frequency does NOT mean that at a given frequency you have a pure sinusoidal tone – that is NOT how noise works.
 - Noise is random and discrete.
 - Therefore, we can only look at **statistics** of noise.
 - We can pass our noise through a **very narrow** ($\pm \delta f$ around center frequency f_c) bandpass filter and measure the total noise power at the output of that filter. – We can listen for a set amount of time, collect the voltages/currents, and calculate the power.
 - This is essentially what we call the **power spectral density** - i.e., the power within a very small bandwidth.
 - We can repeat this measurement by this by moving the center frequency of our bandpass filter so that we have the average noise power at all the different center frequencies f_c from $-\infty$ to $+\infty$.
 - This gives us the power spectral density distribution.

Modelling noise:

Mean

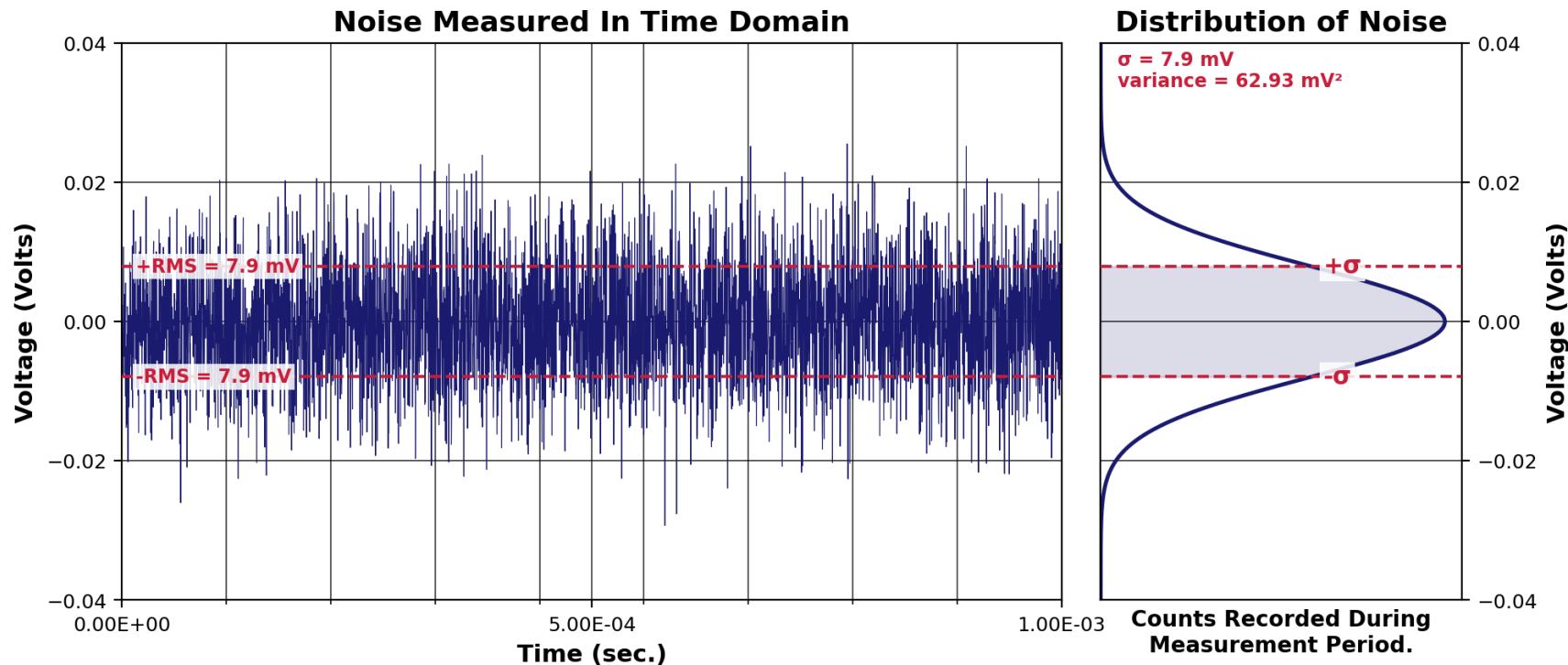
- **Mean** – Generally zero
 - Unit - V or A



Modelling noise:

More about RMS

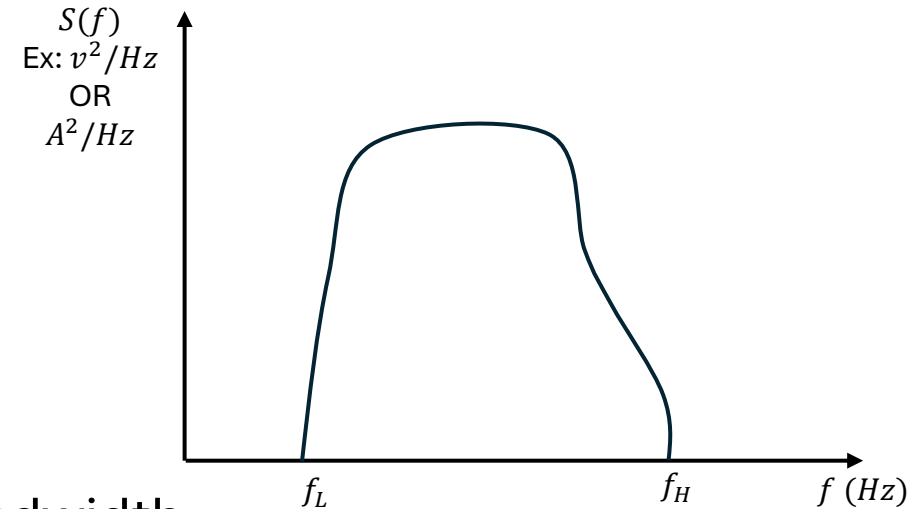
- **Mean** – Generally zero
 - Unit - V or A
- **Variance** – Related to V_{RMS} or I_{RMS} , hence to power.
 - Unit - V^2 or A^2 and proportional to W – Hence, standard deviation is V or A



Modelling noise:

More about PSD

- **Mean** – Generally zero
 - Unit - V or A
- **Variance** – Related to V_{RMS} or I_{RMS} , hence to power.
 - Unit - V^2 or A^2 and proportional to W
 - Hence, standard deviation is V or A
- **Power Spectral Density** – Power the signal carries per unit bandwidth.
 - Denoted as $S(f)$
 - Unit - W/Hz
 - You will also see V^2/Hz or A^2/Hz units.
 - This is commonly called voltage noise power spectral density or current noise power spectral density.
 - In many datasheets, you will also see units of $V/\sqrt{Hz}$ or $A/\sqrt{Hz}$
 - This is commonly referred to as the noise amplitude spectrum density for voltage or current respectively.
 - These are convenient for calculations:
 - Allows direct calculation of noise RMS voltages and currents without the need for ohm's law to go from power back to voltage/current.

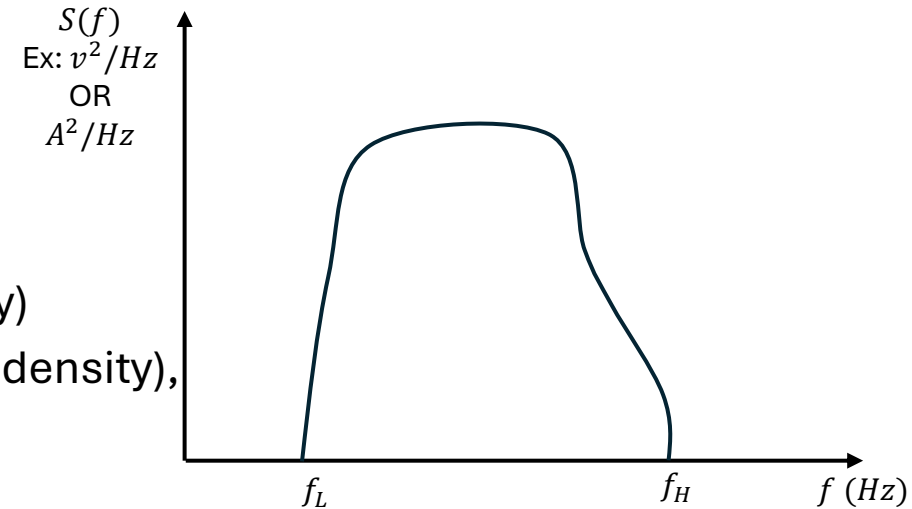


Modelling noise:

More about PSD

- **Power Spectral Density**

- $P_{total} = \int_{f_L}^{f_H} S(f) df$
 - Note: This is if $S(f)$ is given in units of W/Hz (power spectral density)
 - If instead, $S(f)$ was in units of V^2/Hz (voltage noise power spectral density), we get $V_{RMS}^2 = \int_{f_L}^{f_H} S(f) df$
- So, if on the PSD curve, the region between f_L and f_H is flat (Ex: in white noise), then:
 - $P = S(f) \times [f_H - f_L] = S(f) \times B$ where B is the bandwidth.



Noise calculations:

Can we always add noise powers together?

- **NO!** - But, in certain practical circumstances, yes.
- Noise power can only be added together if the noise sources are independent and they have zero mean!
 - Why should they be independent?
 - Imagine there are two noise sources N_1 and N_2 , one generating a certain noise pattern, and the other generating the exact opposite polarity of the same.
 - Now, the total noise power will be zero!
 - They were not independent because one source can be written as a function of the other $N_2 = f(N_1)$, in this case, $N_2 = -N_1$ simply.

Noise calculations:

Can we always add noise powers together?

- More mathematically, $v_{Total}(t) = v_1(t) + v_2(t)$ where these are the sum of the **instantaneous** voltages for sources 1 and 2.
 - Assuming the voltage was across some resistance R the **instantaneous** power would be, $p_T(t) = \frac{(v_1(t)+v_2(t))^2}{R} = \frac{v_1^2(t)}{R} + \frac{2v_1(t)v_2(t)}{R} + \frac{v_2^2(t)}{R}$
 - $\therefore p_T(t) = p_1(t) + p_2(t) + \frac{2v_1(t)v_2(t)}{R}$ where $p_1(t)$ and $p_2(t)$ are the instantaneous powers the two sources would have individually dissipated over the resistance R .
 - Now, if we get the average power by integrating over a window of time T :
 - $P_{AVG} = \frac{1}{T} \int_0^T p_1(t) dt + \frac{1}{T} \int_0^T p_2(t) dt + \frac{2}{TR} \int_0^T v_1(t)v_2(t) dt$
 - $P_{AVG} = P_{1AVG} + P_{2AVG} + \frac{2}{TR} \int_0^T v_1(t)v_2(t) dt$
 - Now, what can we do about this cross term? If we really have all the data points for $v_1(t)$ and $v_2(t)$ over the time window T , we can calculate this.
 - If not, how do we continue?

Noise calculations:

Can we always add noise powers together?

- What if the two noise sources were independent?
 - Theorem: The **mean of the product** between two **independent** random variables becomes equal to the **product of their individual means**.
 - i.e. $E(XY) = E(X)E(Y)$ given X and Y are independent random variables
 - **Note:** this is valid in the limit $T \rightarrow \infty$, where the time-average becomes the statistical expectation $E[\cdot]$ (ergodicity). It is NOT literally splitting $\int_0^T v_1(t)v_2(t) dt$ into $\left(\int_0^T v_1(t) dt\right) \times \left(\int_0^T v_2(t) dt\right)$ but rather, independence lets the expectation itself split; $E[v_1v_2] = E[v_1]E[v_2]$.
 - i.e., as $T \rightarrow \infty$ the $\frac{2}{TR} \int_0^T v_1(t)v_2(t) dt$ term in the formula collapses to
 - $\frac{2}{R} E[v_1v_2] \rightarrow \frac{2}{R} E[v_1]E[v_2]$ where $E[v_1]$ and $E[v_2]$ are the means μ_1 and μ_2 for sources 1 and 2 respectively.
 - Therefore, we have:
 - $P_{AVG} = P_{1AVG} + P_{2AVG} + \frac{2}{R} \mu_1 \times \mu_2$
 - Now, if the **means** of the noise sources are **zero** (which is usually the case), the cross term completely cancels out, giving
 - $P_{AVG} = P_{1AVG} + P_{2AVG}$

Noise calculations:

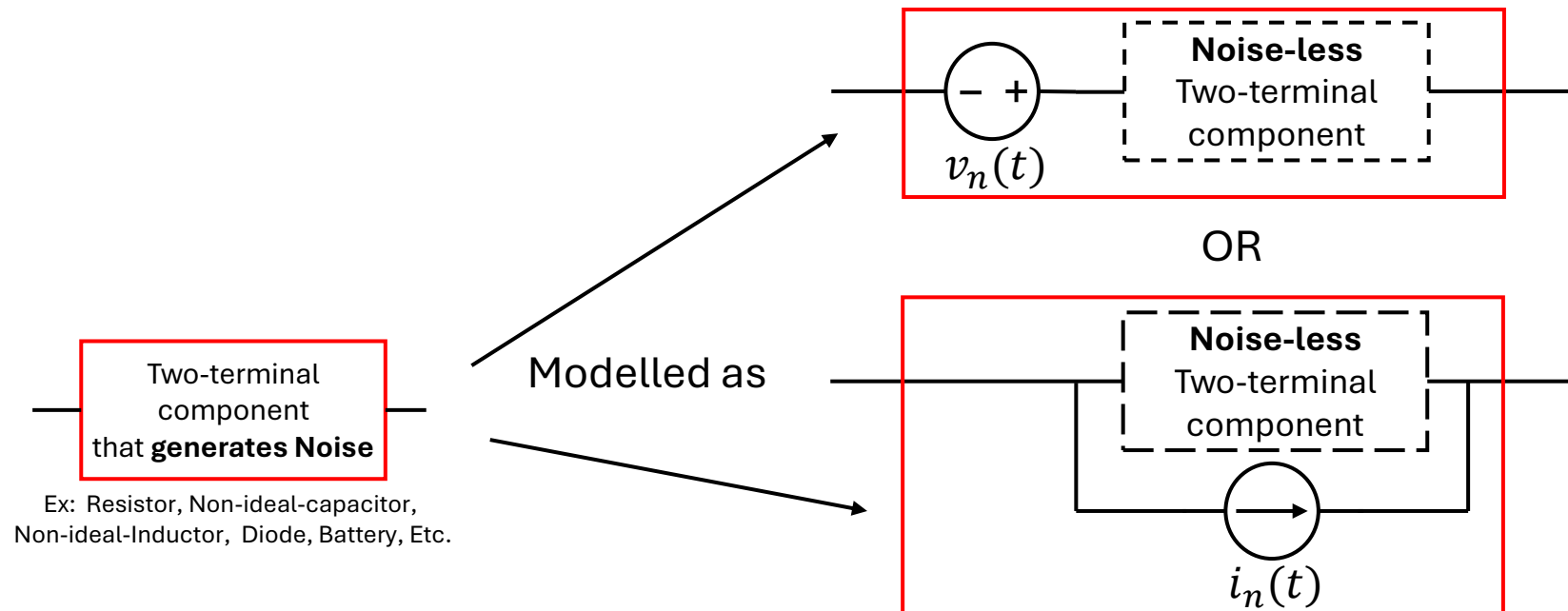
Can we always add noise powers together?

- This is how you arrive at the summation of powers!
 - Therefore, It is important to keep in mind the importance of the **two** conditions required for this to be valid!
- $P_{AVG} = P_{1AVG} + P_{2AVG}$
 - Since $P_{AVG} = \frac{V_{RMS}^2}{R}$, for noise through a resistance R , we get an RMS noise voltage of:

$$\bullet V_{RMS} = \sqrt{R \times P_{AVG}} = \sqrt{R \times (P_{1AVG} + P_{2AVG})} = \sqrt{R \times \left(\frac{V_{1RMS}^2}{R} + \frac{V_{2RMS}^2}{R} \right)} = \sqrt{V_{1RMS}^2 + V_{2RMS}^2}$$

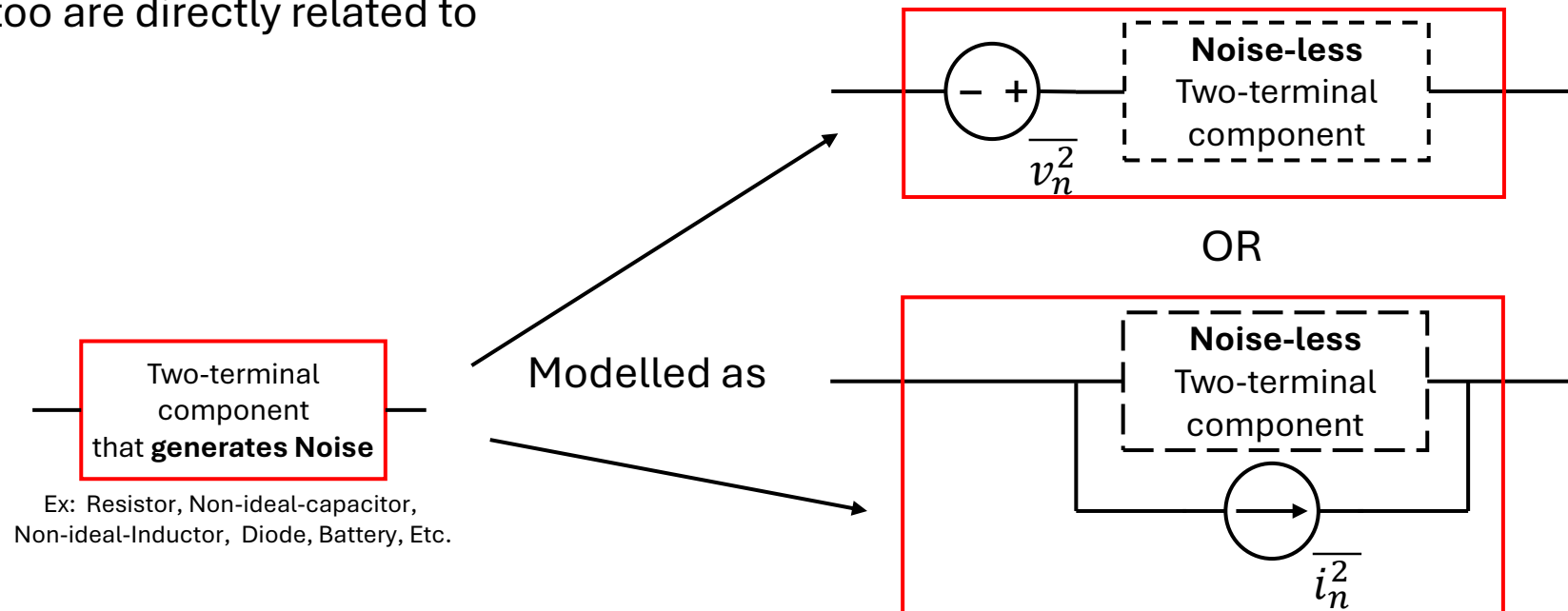
NOTE About Notation

- Remember the following figure?
 - I labeled the voltage/current source as $v_n(t)$ and $i_n(t)$.
 - However, instantaneous values don't make sense during calculations because noise is random.



NOTE About Notation

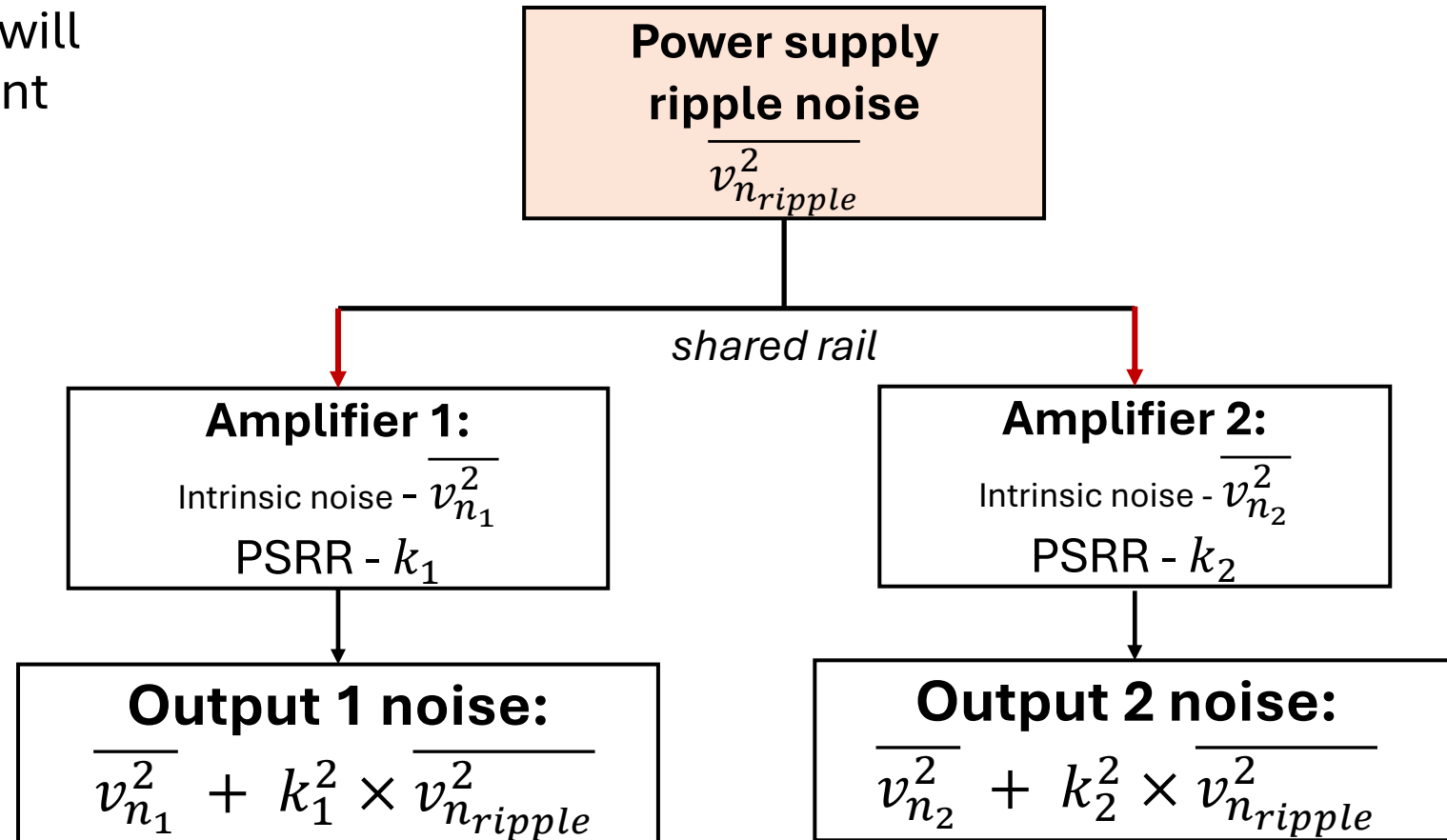
- Remember the following figure?
 - I labeled the voltage/current source as $v_n(t)$ and $i_n(t)$.
 - However, instantaneous values don't make sense during calculations because noise is random.
 - Therefore, in all noise related literature, you will find it resembled as $\overline{v_n^2}$ or $\overline{i_n^2}$.
 - i.e., the Mean squared
 - This is beneficial for calculations because the **voltage/current** noise power spectral densities too are directly related to these $(\overline{v_n^2} / \text{Hz}$ or $\overline{i_n^2} / \text{Hz})$.



When noise sources aren't independent:

Real-world-Example

- Two amplifiers powered from the same noisy power rail both inherit a copy of that same ripple noise.
- This inherited noise is correlated and will appear on top of their own independent noise.



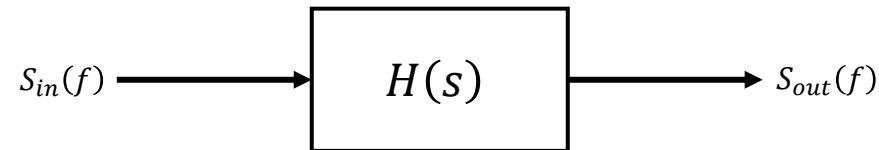
- Both outputs share the SAME $v_{ripple}(t)$ term (via each amp's PSRR k_1 and k_2)
- This ripple noise therefore is correlated.
- However, $\overline{v_{n_1}^2}$ and $\overline{v_{n_2}^2}$ are independent.
- Therefore, the total noise **power** output would be

$$\overline{v_{n_1}^2} + \overline{v_{n_2}^2} + \alpha \overline{v_{n_{ripple}}^2}$$

Where α is some factor that represents the total shared ripple power.

What happens if noise goes through a frequency dependent system?

- A 'system' can be defined via a transfer function $H(s)$.
 - Here, $s = \sigma + j\omega$ is the s-domain
 - If we assume the decay portion σ to be zero, we have $s = j\omega = j2\pi f$.
- Assume we have a noise source with a PSD $S_{in}(f)$.
 - How do we calculate what happens to this noise as it goes through the 'system'?

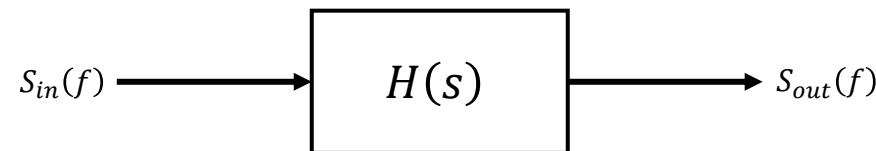


What happens if noise goes through a frequency dependent system?

- From signals and systems theory:

- $S_{out}(f) = |H(j2\pi f)|^2 \times S_{in}(f)$

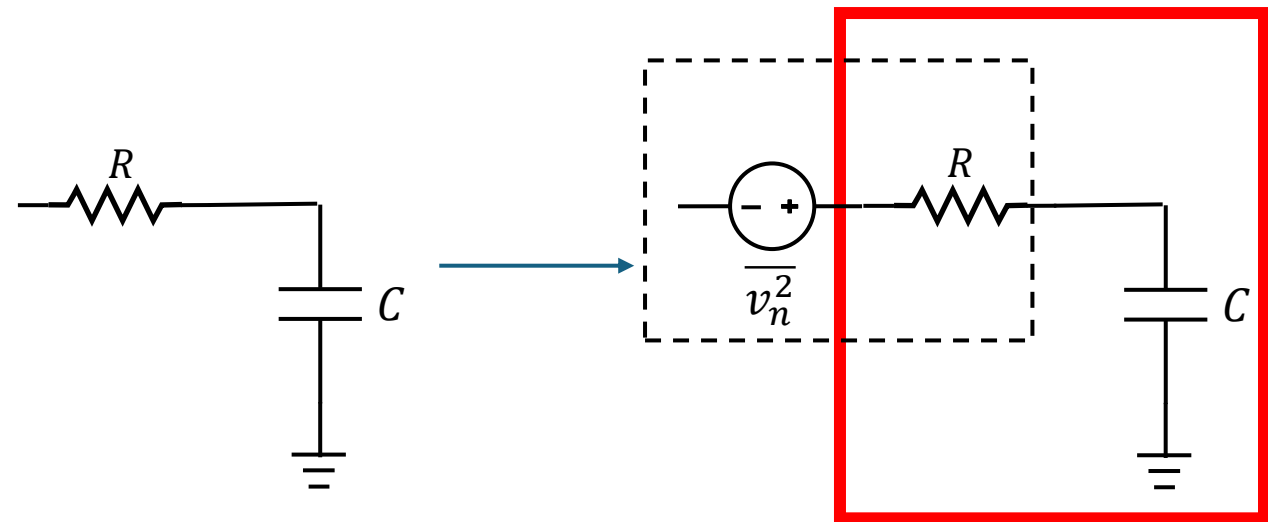
- i.e., if we know the PSD of the noise-source (or any signal) we can calculate its PSD after it goes through the 'system' by simply multiplying the input PSD with the **squared** magnitude of the transfer function.
 - Note: This assumes the system is linear!
 - Note: If the input was the fourier transform of a signal (as opposed to the power spectrum), the formula becomes
 - $v_{out}(s) = H(s) * v_{in}(s)$
 - Hence, remains a complex quantity
 - The power spectral density on the other hand is a power, hence has no phase information.
 - Therefore, it is a real quantity.
- The magnitude of a transfer function may be obtained by multiplying the transfer function by the complex conjugate of itself:
 - $|H(s)|^2 = H(s) \times H(s)^*$



Noise through a frequency dependent system:

Ex: RC-Filter

- Ex: RC filter.
 - Let's begin by considering the capacitor to have zero resistance.
 - An ideal capacitor is noise-less.
 - Therefore, the only noise source is the resistor itself
 - What would be the output noise through the filter?
 - What would you use for the bandwidth of noise?
- We can model the noise in the resistor portion of the RC-network by adding a voltage source in series with the resistor.



Noise through a frequency dependent system:

Ex: RC-Filter

- The portion marked in red becomes a system, with an input v_n^2 .

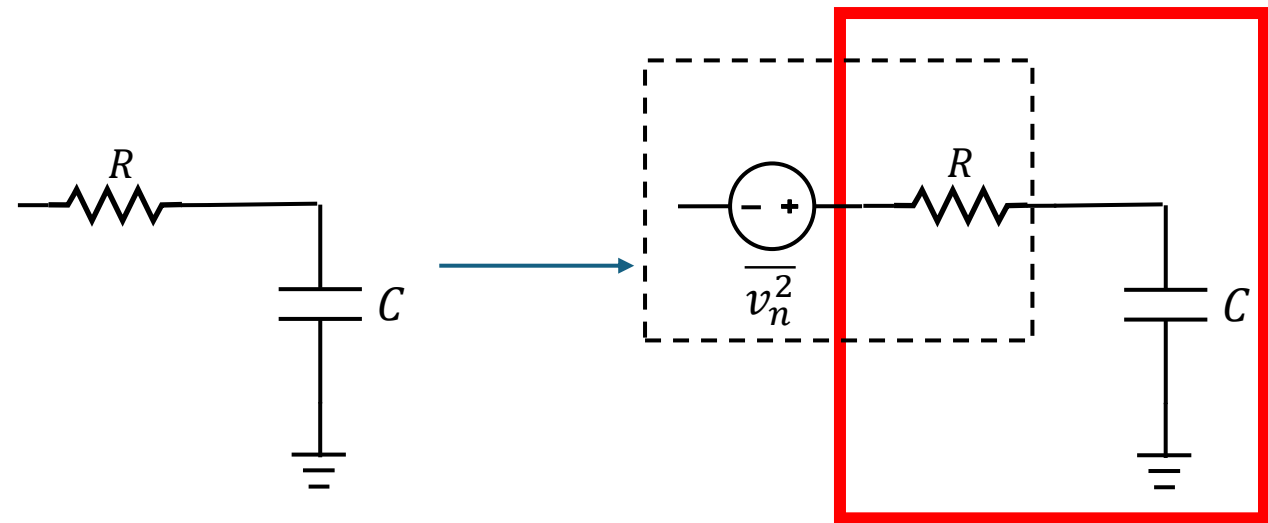
- The transfer function of a RC low pass filter is: $H(s) = \frac{1}{1+j\omega RC}$

- Therefore, the magnitude squared is:

$$|H(s)|^2 = H(s) \times H(s)^* = \frac{1}{1+j\omega RC} \times \frac{1}{1-j\omega RC} = \frac{1}{1+(\omega RC)^2}$$

- So, the PSD of whatever signal that goes in will be multiplied by $|H(s)|^2$.

- $S_{out}(\omega) = \frac{S_{in}(\omega)}{1+(\omega RC)^2} = \frac{S_{in}(f)}{1+(2\pi f RC)^2}$



Noise through a frequency dependent system:

Ex: RC-Filter

- We want to calculate the inherent RMS **voltage** at the output of this filter.
 - Even in the absence of any input signal, there is going to be some voltage purely due to noise from the resistor.
 - We want to calculate $V_{out_{RMS}}$.
- $P_{out_{AVG}} = \frac{V_{out_{RMS}}^2}{R_{load}}$ Here R_{load} is some ideal external load resistance across which we measure the output noise.
- $P_{out_{AVG}} = \int_0^F S_{out}(f) df$
 - Remember, you have to integrate over the power spectral density over all frequencies to get the average power, hence $F \rightarrow \infty$.
- $\therefore P_{out_{AVG}} = \int_0^\infty \frac{S_{in}(f)}{1+(2\pi fRC)^2} df$

Noise through a frequency dependent system:

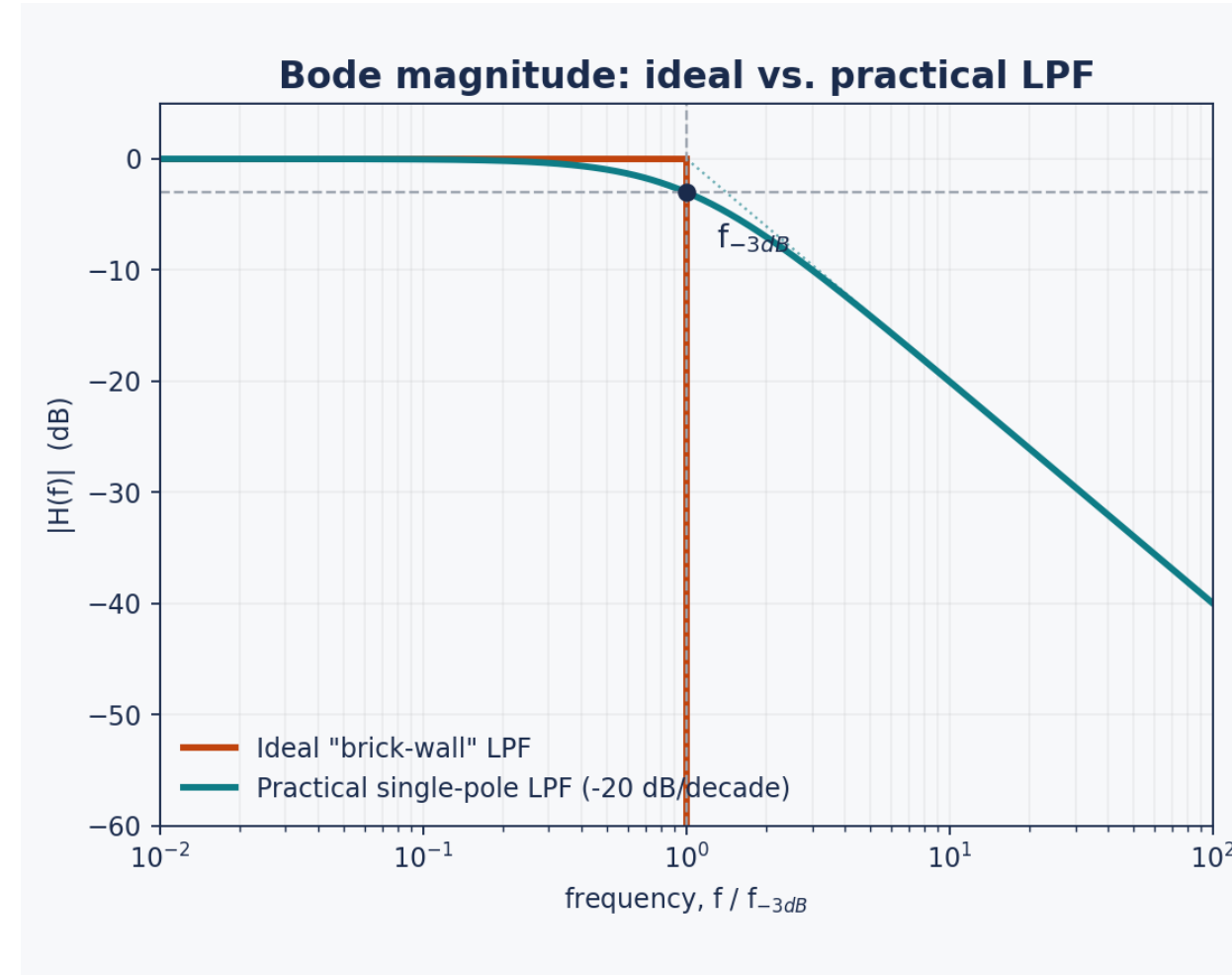
Ex: RC-Filter

- Later, you will learn that the noise in a resistor is called thermal noise, and that thermal noise is a form of white noise.
 - White noise $\Rightarrow$ PSD is flat!
 - i.e., $S_{in}(f)$ is independent of frequency: i.e., a constant S_{in} .
- $\therefore P_{out_{AVG}} = S_{in} \int_0^{\infty} \frac{1}{1+(2\pi fRC)^2} df$
- Now we can solve the integral by substituting $u = 2\pi fRC$
 - $\therefore du = 2\pi RC \times df \Rightarrow df = \frac{du}{2\pi RC}$
 - when $f = 0, u = 0$ and $f \rightarrow \infty, u \rightarrow \infty$
 - $\therefore P_{out_{AVG}} = \frac{S_{in}}{2\pi RC} \int_0^{\infty} \frac{1}{1+u^2} du$
 - $P_{out_{AVG}} = \frac{S_{in}}{2\pi RC} [\arctan(u)]_0^{\infty} = \frac{S_{in}}{2\pi RC} \left[\frac{\pi}{2} - 0 \right]$

Noise through a frequency dependent system:

Ex: RC-Filter

- $\therefore P_{out_{AVG}} = S_{in} \times \frac{1}{2\pi RC} \times \frac{\pi}{2} = \frac{V_{out_{RMS}}^2}{R_{load}}$
- $\therefore V_{out_{RMS}}^2 = R_{load} \times S_{in} \times \frac{1}{2\pi RC} \times \frac{\pi}{2}$
- Now, what is the $\frac{1}{2\pi RC}$ part?
 - It is the -3dB cutoff frequency of a RC low pass filter which we usually denote as f_{-3} .
- $\therefore V_{out_{RMS}}^2 = R_{load} \times S_{in} \times \frac{\pi}{2} f_{-3}$



Noise through a frequency dependent system:

Ex: RC-Filter

- Therefore, for an RC filter, the RMS noise voltage across the capacitor is:
 - $\therefore V_{out_{RMS}}^2 = R_{load} \times S_{in} \times \frac{\pi}{2} f_{-3}$
- In a practical RC filter, although we consider the cutoff frequency to be at the -3dB point, that is only for convenience.
 - That is the frequency where the power of the signal going through the filter attenuates by half its original value.
 - This does **NOT** mean that the rest of the frequencies are totally blocked – they pass too, but with greater and greater attenuation as frequency increases.

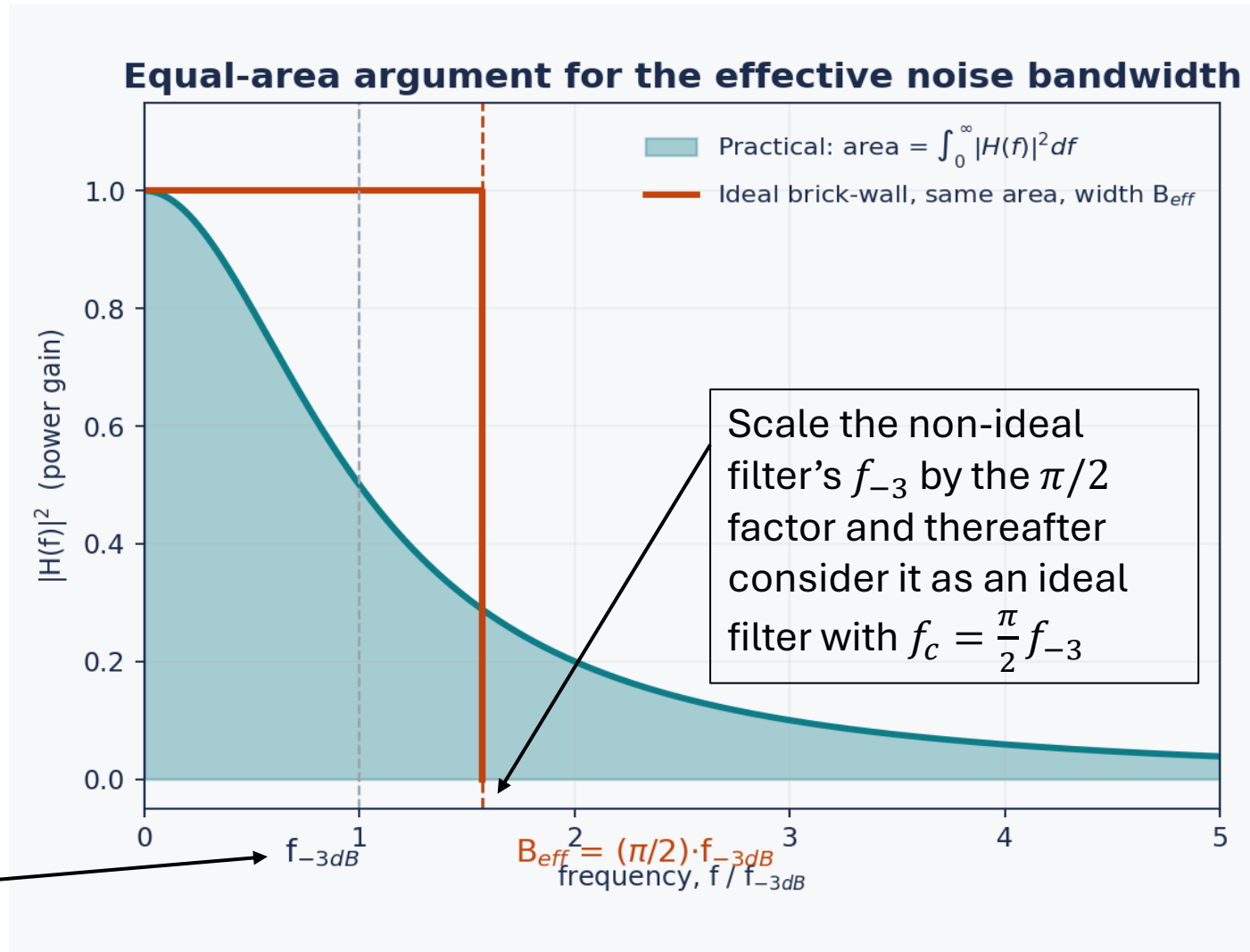
Noise through a frequency dependent system:

Ex: RC-Filter

- Therefore, for a practical RC low pass filter that allows higher frequencies than the nominal cutoff frequency to pass in the roll-off region, the total noise power that passes through (including the portion in the roll-off region) can be obtained by considering it as a brick-wall-filter (ideal RC low pass filter) with an ideal cutoff frequency of $\frac{\pi}{2} \times f_{-3}$ where f_{-3} is the -3dB cutoff frequency of the actual non-ideal filter.
- This $\frac{\pi}{2} f_{-3}$ factor is known as the effective bandwidth.

Noise through a frequency dependent system:

Ex: RC-Filter



Noise Figure

Introduction

- Quantifies how much a system degrades the SNR of a signal passing through it due to the additional noise introduced by the system.
 - The noise figure enables comparison between devices (as long as they are referenced to the same source impedance).
- Noise Figure (NF) is given in decibel units and is obtained by converting a quantity called the Noise Factor (F) into decibel.
 - Noise Figure = $10 \log_{10}$ Noise Factor
- The noise factor F is defined as:
 - $F = \frac{\text{SNR of Input Signal}}{\text{SNR of Output Signal}} = \frac{\text{SNR}_{\text{in}}}{\text{SNR}_{\text{out}}}$
Where $\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{P_S}{P_N}$
 - We have the input and output SNRs as: $\text{SNR}_{\text{IN}} = \frac{P_{S_{\text{in}}}}{P_{N_{\text{in}}}}$ and $\text{SNR}_{\text{OUT}} = \frac{P_{S_{\text{out}}}}{P_{N_{\text{out}}}}$
 - $\therefore F = \frac{\left(\frac{P_{S_{\text{in}}}}{P_{N_{\text{in}}}}\right)}{\left(\frac{P_{S_{\text{out}}}}{P_{N_{\text{out}}}}\right)} = \frac{P_{S_{\text{in}}} \times P_{N_{\text{out}}}}{P_{N_{\text{in}}} \times P_{S_{\text{out}}}}$

Noise Figure

The Input Noise SNR

- To calculate the noise factor, we first need the input noise SNR:
 - $SNR_{in} = \frac{P_{S_{in}}}{P_{N_{in}}}$
 - For that, we need the input noise power.
 - An actual signal will have some inherent noise on top of it due to interference, noise from previous stages, etc.
 - Although this is what is generally considered as signal noise, this is **NOT** what we want here.
 - We want something more standardized as the input noise power.
 - All signal sources have source resistance, hence thermal noise.
 - The amount of this noise power that gets transferred into a system depends both on this source resistance R_S , AND the input resistance R_L of the receiving system.
 - However, the maximum power transferrable this way is given via the maximum power transfer theorem.
 - This maximum power evaluates to $k_B T \Delta f$.
 - Conventionally, T is selected to be 290 Kelvin, thereby giving a **standardized input noise reference value**.

Noise Figure

The Standardized Input Noise Reference

- The standardized input noise reference can be calculated as follows:
 - Consider a voltage source v_s delivering current i_s to a load R_L through a source resistance R_S
 - Since $v_s = i(R_S + R_L)$,
 - the power transferred to the load is: $P_L = i^2 R_L = \frac{v_s^2 R_L}{(R_S + R_L)^2}$
 - Maximum power is transferred when $R_L = R_S$ (matched load),
 - delivering $P_{L_{Max}} = \frac{v_s^2 R_S}{(R_S + R_S)^2} = \frac{v_s^2}{4R_S}$
 - Since we considered the input noise to be sourced by the thermal noise from the input resistance, we have $v_s^2 = 4k_B T \Delta f R_S$
 - Therefore, the maximum power transferrable to any load (hence the input impedance of any system/device) is:
 - $P_{L_{Max}} = \frac{v_s^2}{4R_S} = \frac{4k_B T \Delta f R_S}{4R_S} = k_B T \Delta f$
 - Although the thermal noise voltage and current depended on the source impedance, the noise power itself did NOT, hence became independent of the source characteristics, making this an ideal noise floor for the input noise power for cross-device comparison.
- Therefore $P_{N_{in}} = k_B T \Delta f$ where $T=290$ K.

Noise Figure

The Output Noise SNR

- The output noise is simply the intrinsic noise (P_{N_i}) of the system **plus** the input noise $P_{N_{in}}$ scaled by the power gain G of the system.
 - i.e., $P_{N_{out}} = P_{N_i} + P_{N_{in}} \times G$
 - Note: If the intrinsic noise of the system is also given as an **input referred quantity** ($P_{N_{i-ir}}$) (which is usually the case), it must also be scaled by the power gain G to get its contribution to output noise power.
 - $P_{N_{out}} = (P_{N_{in}} + P_{N_{i-ir}}) \times G$
- Similarly, the power of the output **signal** ($P_{S_{out}}$) is simply the power of the input signal ($P_{S_{in}}$) scaled by the power gain G of the system/device.
 - i.e., $P_{S_{out}} = P_{S_{in}} \times G$

Noise Figure

Noise Factor Simplified

- Therefore, the Noise Factor simplifies to:

- $$F = \frac{P_{S_{in}} \times P_{N_{out}}}{P_{N_{in}} \times P_{S_{out}}} = \frac{P_{S_{in}} \times (P_{N_{in}} + P_{N_{i-ir}}) \times G}{P_{N_{in}} \times P_{S_{in}} \times G} = \frac{P_{N_{in}} + P_{N_{i-ir}}}{P_{N_{in}}} = 1 + \frac{P_{N_{i-ir}}}{P_{N_{in}}}$$

- $$\therefore F = 1 + \frac{\text{Intrinsic Noise Power}}{\text{Noise power of a standardized input impedance (Reference Noise Power)}}$$

- Since the standard input noise power = $k_B \times 290\Delta f$:

- $$F = 1 + \frac{\text{Intrinsic Noise Power}}{k_B \times 290 \times \Delta f}$$

- We can see how the input **signal** power ($P_{S_{in}}$) and the power gain G disappears from the final formula.
 - Therefore, the noise factor is independent of the signal itself and is only dependent on the intrinsic noise that the system adds.
 - This enables the use of the noise factor (hence noise figure) to compare the noise performance between different systems.

Noise Figure

- We represent the Noise Figure as a decibel version of the Noise Factor:
 - $\therefore NF = 10\log_{10}(F)$
 - $\Rightarrow NF = 10\log_{10}\left(1 + \frac{\text{Intrinsic Noise Power}}{k_B \times 290 \times \Delta f}\right)$ OR $NF = 10\log_{10}\left(1 + \frac{\text{Intrinsic Noise Power Spectral Density}}{k_B \times 290}\right)$
 - A noiseless device or system would therefore have a noise factor of 1, or a noise figure of 0 dB.
 - Since all practical systems add noise, their noise factors are generally greater than 1.
 - The closer a device's noise factor is to 1, the better it is.
- Although the source impedance cancels out in the **standardized input reference noise** derivation, this does **NOT** mean that the noise factor/figure itself is independent of the source impedance.
 - For a two-port device, the input referred intrinsic noise contains an equivalent voltage noise source **and** current noise source.
 - The amount of actual noise power the device will introduce depends on the source impedance.
 - For example, for a voltage amplifier system, if the source has zero impedance, the input referred current noise will have no impact because it will **NOT** generate any noise voltage at the input when it goes through a $0\ \Omega$ resistor.
 - If on the other hand, the source has a very large impedance, the input referred current noise will dominate by inducing a large noise voltage as current noise flows through this large source resistance.
 - Therefore, we will later see that the noise figure can be used as a figure of comparison between devices **ONLY** if they are evaluated for the **same** source impedance.