



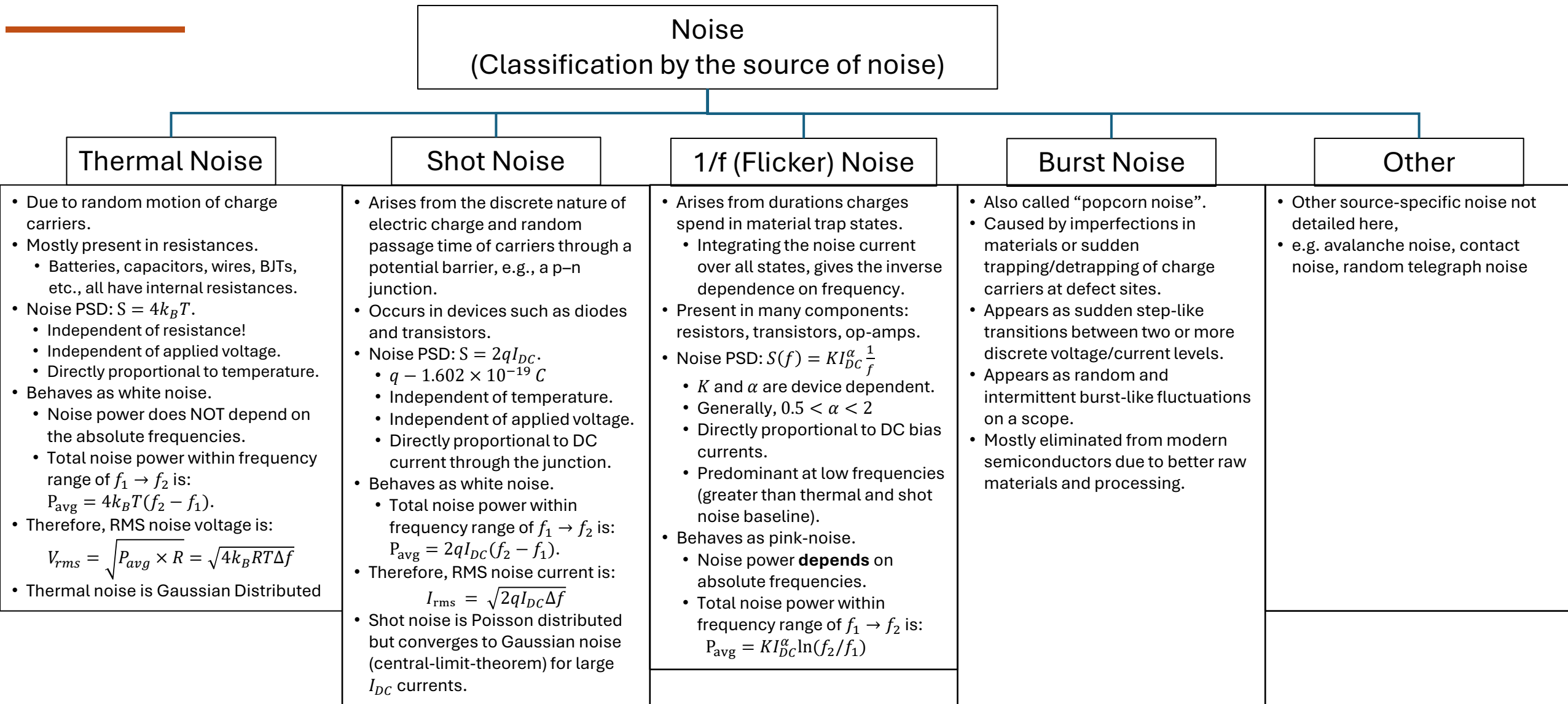
University of Moratuwa
Wisdom is all wealth

Noise and Interference in Instrumentation

Part 2: Classification of Noise

Anton F. P. Van Putte, Electronic measurement systems, Taylor & Francis, 2nd Edition
M. M. S. Anand, Electronic instruments and Instrumentation Technology, PHI

Classification of Noise: By physics behind its origin



Thermal Noise – Basics

- Also known as Johnson-Nyquist noise or simply Johnson noise.
 - Caused by random motion of charge carriers due to thermal excitation.
- Present in all resistances.
 - NOTE: Batteries, capacitors, wires, diodes, transistors, etc., all have internal resistances.
- Characteristics:
 - For a given resistance, Noise PSD: $S = 4k_B T$
 - where k_B is the Boltzmann constant ($1.381 \times 10^{-23} J/K$)
 - and T is the absolute temperature in Kelvin.
 - Amount of noise **independent** of **applied voltage**.
 - i.e., whether or not you apply external voltage to a resistance, you will still have this noise.
 - Noise power directly **proportional** to **temperature**.

Thermal Noise – Basics

- Characteristics continued:
 - Spectrally uniform – **white noise**
 - Noise power does NOT depend on the **absolute** values of frequencies that we allow into our circuit
 - It only depends on the total bandwidth.
 - i.e., the integral in the spectral density function to power conversion formula just becomes an integral over a constant!
 - Therefore, total noise power within frequency range of $f_1 \rightarrow f_2$ is:
 - $P_{avg} = 4k_B T(f_2 - f_1)$.
 - Therefore, RMS noise voltage is:
 - $V_{rms} = \sqrt{P_{avg} \times R} = \sqrt{4k_B R T \Delta f}$

Thermal Noise – Single Resistor

- Calculate thermal noise **voltage** (RMS) generated by a resistor $R=10\text{ k}\Omega$ at room temperature (25°C).
 - Anything missing?

Thermal Noise – Single Resistor

- Calculate thermal noise **voltage** (RMS) generated by a resistor $R=10\text{ k}\Omega$ at room temperature (25°C).
 - Anything missing?
 - We need the bandwidth!
 - Assume we filter out all frequencies except between 1 kHz and 11 kHz.

Thermal Noise – Single Resistor

- $S = 4k_B T$
 - with $k_B = 1.381 \times 10^{-23} \text{ J/K}$, $T = 25^\circ\text{C} = 298.15 \text{ K}$
- Plugging in;
 - $4k_B T = 4 \times 1.381 \times 10^{-23} \text{ J/K} \times 298.15 \text{ K} = \mathbf{1.647 \times 10^{-20} \text{ W/Hz}}$
- Since $P = \int_{F_L}^{F_H} S(f) df$ and thermal noise is white, the integration converts to simply multiplying by bandwidth.
 - $P = S \times \Delta f = 1.647 \times 10^{-20} \text{ W/Hz} \times (11 \times 10^3 - 1 \times 10^3) \text{ Hz} = \mathbf{1.647 \times 10^{-16} \text{ W}}$
- Since $V_{RMS} = \sqrt{P \times R} \rightarrow V_{RMS} = \sqrt{1.647 \times 10^{-16} \text{ W} \times 10^4 \Omega} = \mathbf{1.28 \mu V}$
 - That is NOT a small number!
- Equivalent current-noise is $I_{RMS} = \sqrt{\frac{P}{R}} = \sqrt{\frac{1.647 \times 10^{-16} \text{ W}}{10^4 \Omega}} = \mathbf{128 \text{ pA}}$

Thermal Noise – Single Resistor

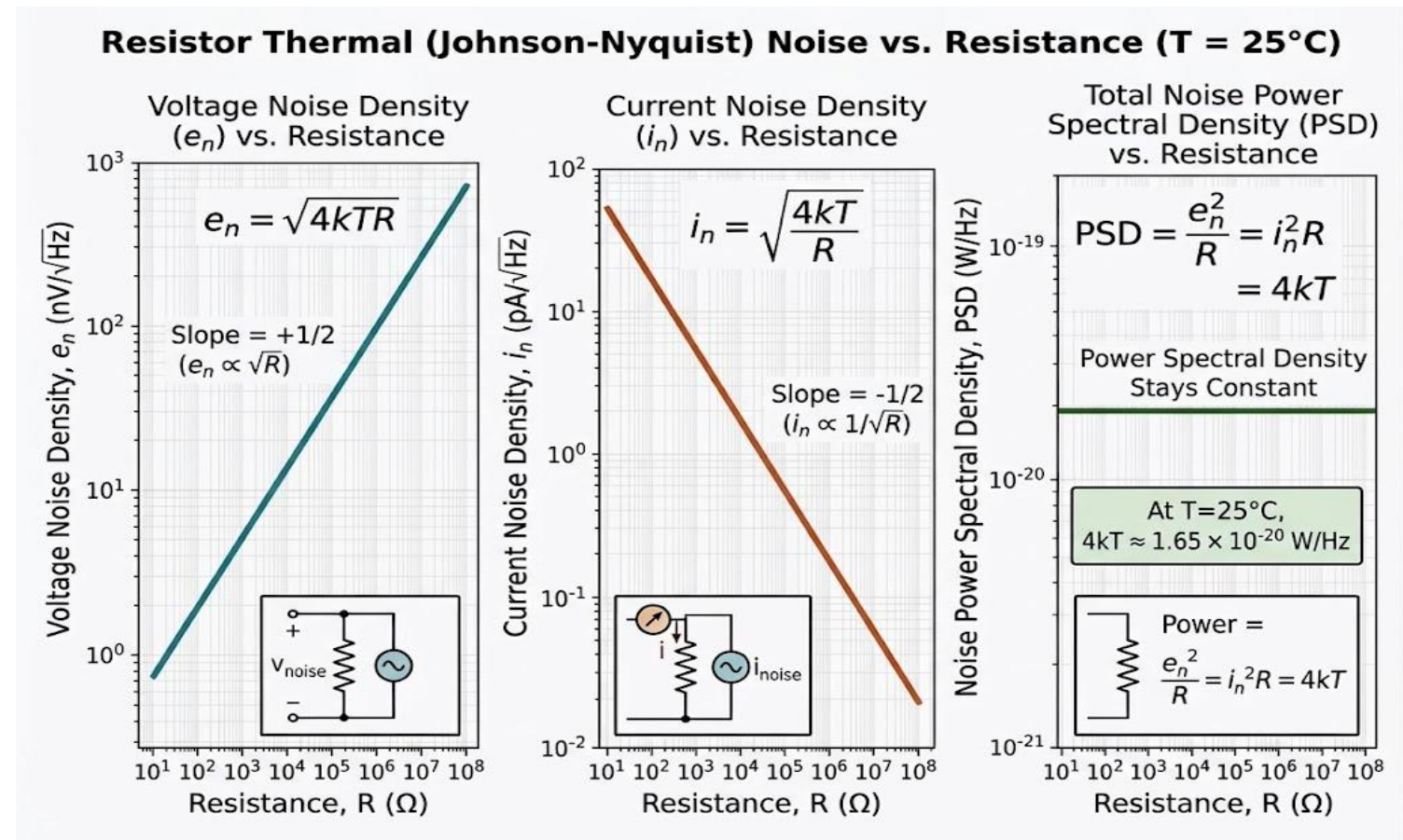
- Tricky questions:
 - What if you change the bandwidth to be between 20 kHz and 30 kHz instead?
 - What if you change the bandwidth to be between 20 MHz and 30 MHz instead?

HOMEWORK!

Thermal Noise – Single Resistor

Noise vs Resistance

- For a set temperature, how does the noise (voltage, current, power) generated by a resistor change with change in resistance?



Thermal Noise – Single Resistor

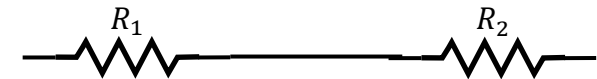
Noise vs Resistance

- For a set temperature, how does the noise generated by a resistor change with resistance?
 - For a resistor,
 - Noise **voltage increases** with increase in resistance.
 - Noise **current decreases** with increase in resistance.
 - Noise **power is independent** of resistance.
 - You will later realize why this is important to understand.

Thermal Noise – Two Resistors in Series

Total Current Noise

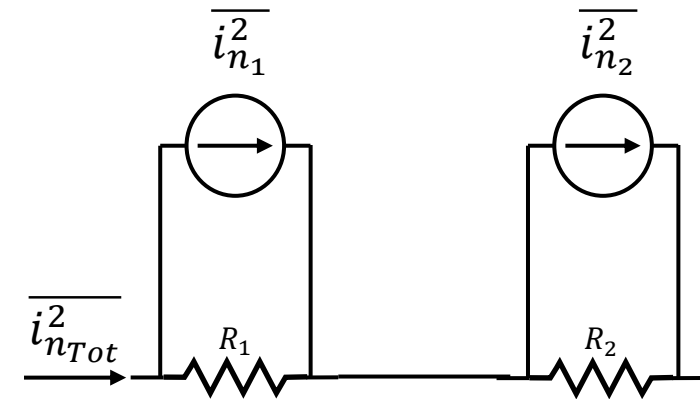
- Calculate the total RMS noise **current** generated by two resistors with resistance R_1 and R_2 wired in series. Assume we measure the noise through an ideal bandpass filter with the pass-band between f_L and f_H Hz (hence a bandwidth $\Delta f = f_H - f_L$) and that resistor 1 is at temperature T_1 and resistor 2 is at temperature T_2 .
 - A single resistor generates thermal noise with PSD:
 - $S = 4k_B T$
 - Therefore, noise power of a single resistor is:
 - $P = 4k_B T \Delta f$
 - Noise current through a single resistor is:
 - $I_{RMS} = \sqrt{\frac{4k_B T \Delta f}{R}}$
 - How to extend this to two resistors in series?



Thermal Noise – Two Resistors in Series

Total Current Noise

- We can use our 2-terminal (1-port) model of noise for **each** resistor, with the current through each being $\overline{i_{n_1}^2} = \frac{4k_B T_1 \Delta f}{R_1}$ and $\overline{i_{n_2}^2} = \frac{4k_B T_2 \Delta f}{R_2}$.
- Next, we find the total noise current $\overline{i_{n_{Tot}}^2}$
 - We know that:
 - Mean of thermal noise is zero.
 - Two different resistors generate noise independently; hence the noise signals are uncorrelated.
 - Therefore, we should be able to add the noise **powers** together!
 - $P_{tot} = P_1 + P_2 \Rightarrow \therefore P_{tot} = \frac{\overline{i_{n_1}^2}}{R_1} + \frac{\overline{i_{n_2}^2}}{R_2}$
 - But this does **NOT** help us with the total current.
 - How do we calculate the **total** noise **current**?
 - Can we just add $\overline{i_{n_1}^2}$ and $\overline{i_{n_2}^2}$?
 - No! Direct addition of currents is only possible when sources are parallel!



Thermal Noise – Two Resistors in Series

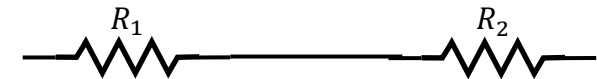
Total Current Noise

- Now you will realize that this calculation is difficult.
 - To find $\overline{i_{n_{Tot}}^2}$ you will have to consider one source at a time, setting the other current source to open circuit, calculating the current contribution by that source, repeat for the next source, then add the contributions together.
 - $\overline{i_{n_1}^2} |_{i_{n_2} \text{ open}} = \overline{i_{n_1}^2} \cdot \left(\frac{R_1}{R_1 + R_2} \right)^2$
 - $\overline{i_{n_2}^2} |_{i_{n_1} \text{ open}} = \overline{i_{n_2}^2} \cdot \left(\frac{R_2}{R_1 + R_2} \right)^2$
 - $\therefore \overline{i_{n_{Tot}}^2} = \overline{i_{n_1}^2} |_{i_{n_2} \text{ open}} + \overline{i_{n_2}^2} |_{i_{n_1} \text{ open}} = \overline{i_{n_1}^2} \cdot \left(\frac{R_1}{R_1 + R_2} \right)^2 + \overline{i_{n_2}^2} \cdot \left(\frac{R_2}{R_1 + R_2} \right)^2$
 - Since $\overline{i_{n_1}^2} = \frac{4k_B T_1 \Delta f}{R_1}$ and $\overline{i_{n_2}^2} = \frac{4k_B T_2 \Delta f}{R_2}$:
 - $\therefore \overline{i_{n_{Tot}}^2} = \frac{4k_B T_1 \Delta f}{R_1} \cdot \left(\frac{R_1}{R_1 + R_2} \right)^2 + \frac{4k_B T_2 \Delta f}{R_2} \cdot \left(\frac{R_2}{R_1 + R_2} \right)^2 = \frac{4k_B \Delta f}{(R_1 + R_2)^2} \cdot (T_1 R_1 + T_2 R_2)$
 - This was tedious and unnecessary!

Thermal Noise – Two Resistors in Series

Total Current Noise

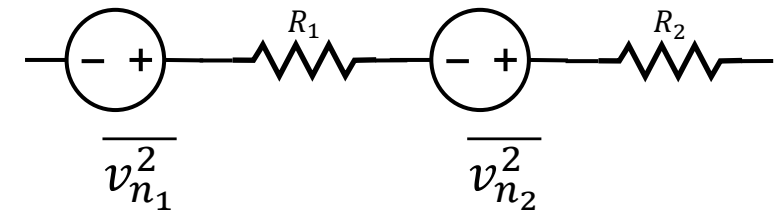
- This calculation became difficult because you can only add two currents together to get the total current when both currents enter/exit into the same branch!
- We could have easily avoided this hassle had we used Thevenin's form instead!
 - Voltages always add in series!



Thermal Noise – Two Resistors in Series

Total Current Noise

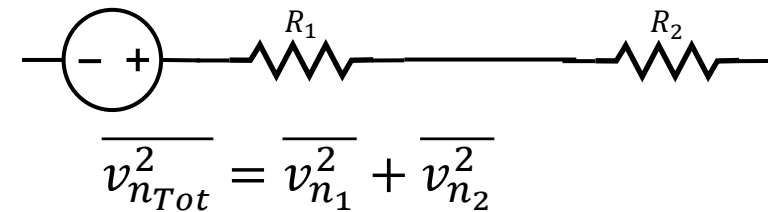
- Therefore, if we repeat the calculation with the series voltage sources model:
 - $v_{n_1}^2 = 4k_B T_1 \Delta f R_1$ and $v_{n_2}^2 = 4k_B T_2 \Delta f R_2$
 - Since we know that two voltage sources in series can be simply represented as their sum:



Thermal Noise – Two Resistors in Series

Total Current Noise

- Therefore, if we repeat the calculation with the series voltage sources model:
 - $v_{n_1}^2 = 4k_B T_1 \Delta f R_1$ and $v_{n_2}^2 = 4k_B T_2 \Delta f R_2$
 - We know that two voltage sources in series can be replaced by a single source whose voltage is given by the sum of the individual voltages: $v_{n_{Tot}}(t) = v_{n_1}(t) + v_{n_2}(t)$
 - Therefore, the total mean square noise $\overline{v_{n_{Tot}}^2} = \overline{(v_{n_1}(t) + v_{n_2}(t))^2} = \overline{v_{n_1}^2} + \overline{v_{n_2}^2}$
 - As we did in slides 17-20, by exercising the independence and zero mean statistics of the two voltage noise variables, we can get rid of the cross term to obtain $\overline{v_{n_{Tot}}^2} = \overline{v_{n_1}^2} + \overline{v_{n_2}^2}$
 - $\therefore \overline{v_{n_{Tot}}^2} = 4k_B T_1 \Delta f R_1 + 4k_B T_2 \Delta f R_2 = 4k_B \Delta f (T_1 R_1 + T_2 R_2)$



Thermal Noise – Two Resistors in Series

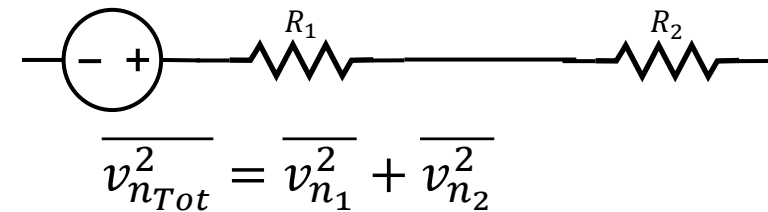
Total Current Noise

- Since noise current between both resistors are the same, we can use Ohm's law:
 - Remember, because we are working with the squared RMS values, each term in the ohm's law equation must be squared.

$$\bullet \overline{i_{n_{Tot}}^2} = \frac{\overline{v_{n_{Tot}}^2}}{\left(\text{Total Resistance across which } \overline{v_{n_{Tot}}^2} \text{ spreads}\right)^2}$$

$$\bullet \therefore \overline{i_{n_{Tot}}^2} = \frac{4k_B\Delta f(T_1R_1 + T_2R_2)}{(R_1 + R_2)^2}$$

- The same as what we got when we used the current sources, but much easier calculation.



Thermal Noise – Two Resistors in Series

Total Current Noise

- Now, if we assume both resistors were at the same temperature $T_1 = T_2 = T$,

- $V_{n_{TotRMS}} = \sqrt{v_{n_{Tot}}^2} = \sqrt{4k_B T \Delta f (R_1 + R_2)}$

- $i_{n_{TotRMS}} = \sqrt{i_{n_{Tot}}^2} = \sqrt{\frac{4k_B T \Delta f}{R_1 + R_2}}$

- Essentially, if the **temperature** ($T_1, T_2, \dots$) of **multiple resistors** ($R_1, R_2, \dots$) is the **same**, the total noise voltage/current when they are wired in **series** acts as if they were generated by **one lumped resistor** with resistance $R_{Series} = R_1 + R_2 + \dots$

$$\therefore V_{n_{TotRMS}} = \sqrt{4k_B T \Delta f R_{Series}}$$

and $I_{n_{TotRMS}} = \sqrt{\frac{4k_B T \Delta f}{R_{Series}}}$

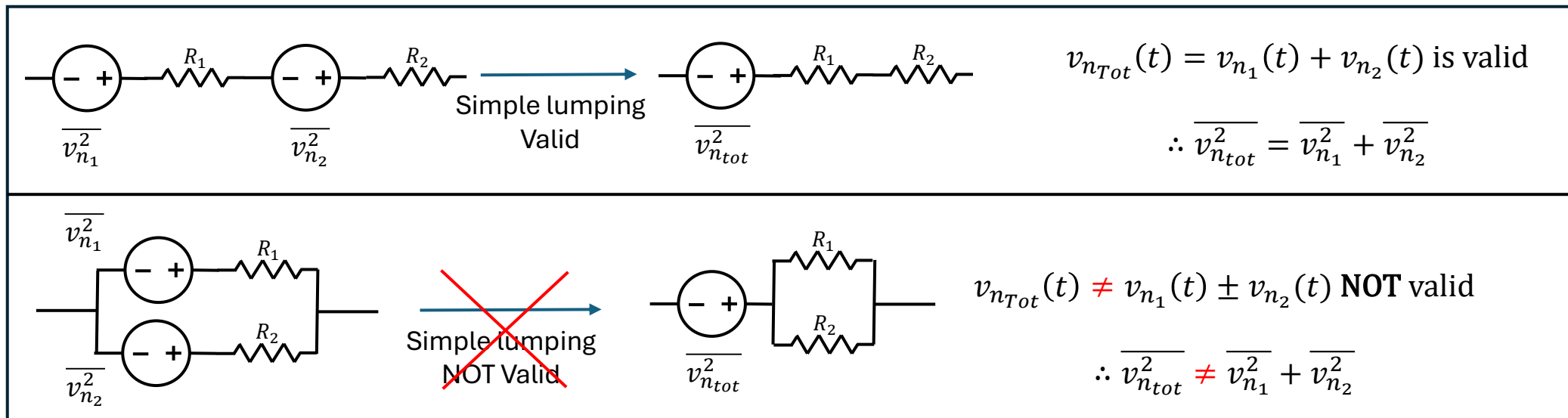
IMPORTANT: About adding noise voltages/currents

- We learned that $P_{tot} = P_1 + P_2 + \dots$
 - If mean noise voltage/current is zero and the noise voltage/current sources are uncorrelated.
- In the previous calculation, we saw that this is also true for **any** two (or more) random variables that have zero mean, and are uncorrelated, as long as their time domain addition is valid!
 - For **voltage sources** this is valid only if they are in **series** with each other:
 - i.e., $\overline{v_{tot}^2} = \overline{v_1^2} + \overline{v_2^2}$ because $v_{tot}(t) = v_1(t) \pm v_2(t)$ is valid for series voltages (KVL).
 - For **currents sources**, this is only valid if they are parallel to each other:
 - i.e., $\overline{i_{tot}^2} = \overline{i_1^2} + \overline{i_2^2}$ because $i_{tot}(t) = i_1(t) \pm i_2(t)$ is valid for currents at a single node (KCL).

IMPORTANT: About adding noise voltages/currents:

Voltages

- Noise **voltage mean square (NOT RMS)** values can be added if:
 - The noise means are zero.
 - The noise sources are uncorrelated.
 - Thevenin is applicable: Voltage sources are in series.
- However, voltage **RMS** values can **NOT** be added directly in any situation.
 - $V_{TotRMS} \neq V_{1RMS} + V_{2RMS}$ because $V_{TotRMS} = \sqrt{v_{Tot}^2} = \sqrt{v_{n_1}^2 + v_{n_2}^2}$ and **NOT** $\sqrt{v_{n_1}^2} + \sqrt{v_{n_2}^2}$

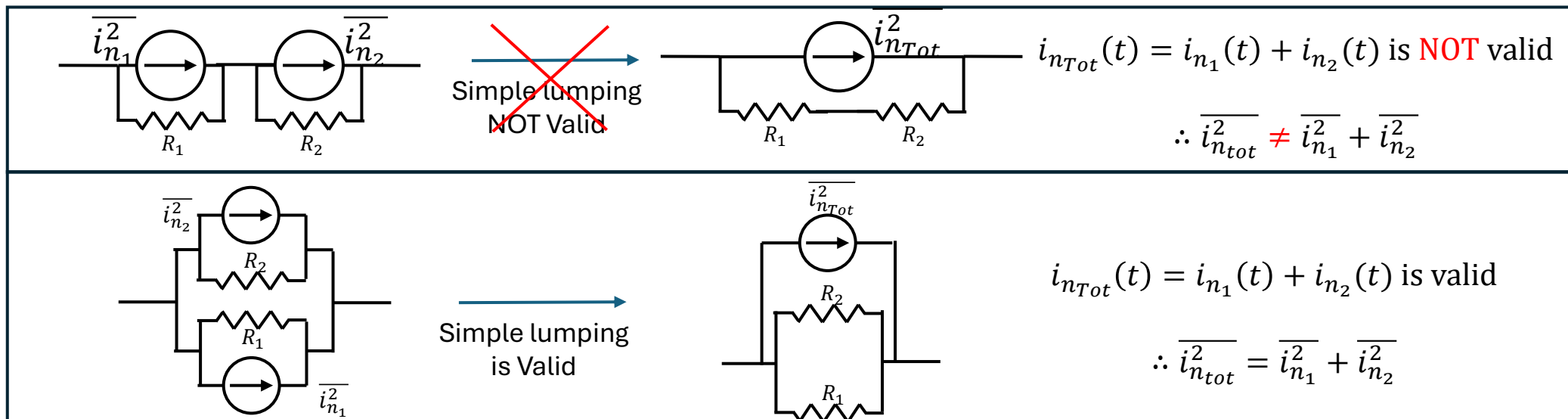


IMPORTANT: About adding noise voltages/currents:

Currents

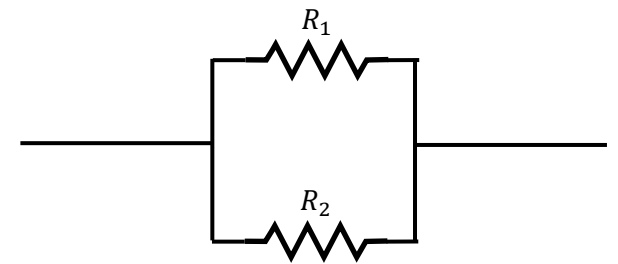
- Noise **current mean square (NOT RMS)** values can be added if:
 - The noise means are zero.
 - The noise sources are uncorrelated.
 - Norton is applicable: Current sources are in parallel.
- However, current **RMS** values can **NOT** be added directly in any situation.

- $I_{TotRMS} \neq I_{1RMS} + I_{2RMS}$ because $I_{TotRMS} = \sqrt{\overline{i_{Tot}^2}} = \sqrt{\overline{i_{n_1}^2} + \overline{i_{n_2}^2}}$ and **NOT** $\sqrt{\overline{i_{n_1}^2}} + \sqrt{\overline{i_{n_2}^2}}$



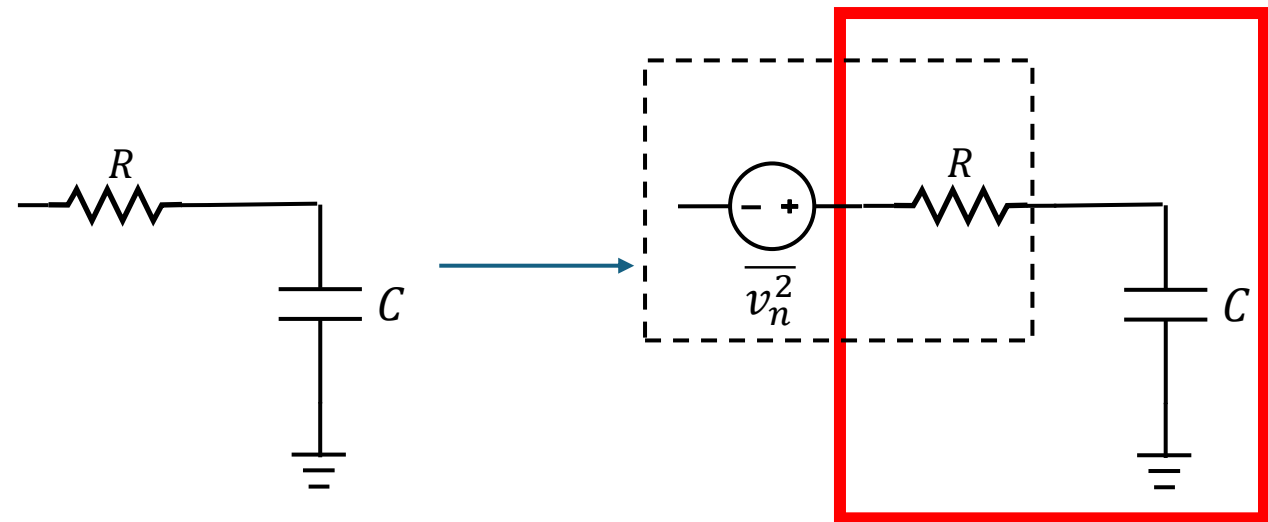
Thermal Noise – Two Resistors in **Parallel**

- Calculate the total RMS noise **voltage** and **current** generated by two resistors with resistance R_1 and R_2 wired in **parallel**. Assume we measure the noise through an ideal bandpass filter with the pass-band between f_L and f_H Hz (hence a bandwidth $\Delta f = f_H - f_L$) and that resistor 1 is at temperature T_1 and resistor 2 is at temperature T_2 .
- HOMEWORK!



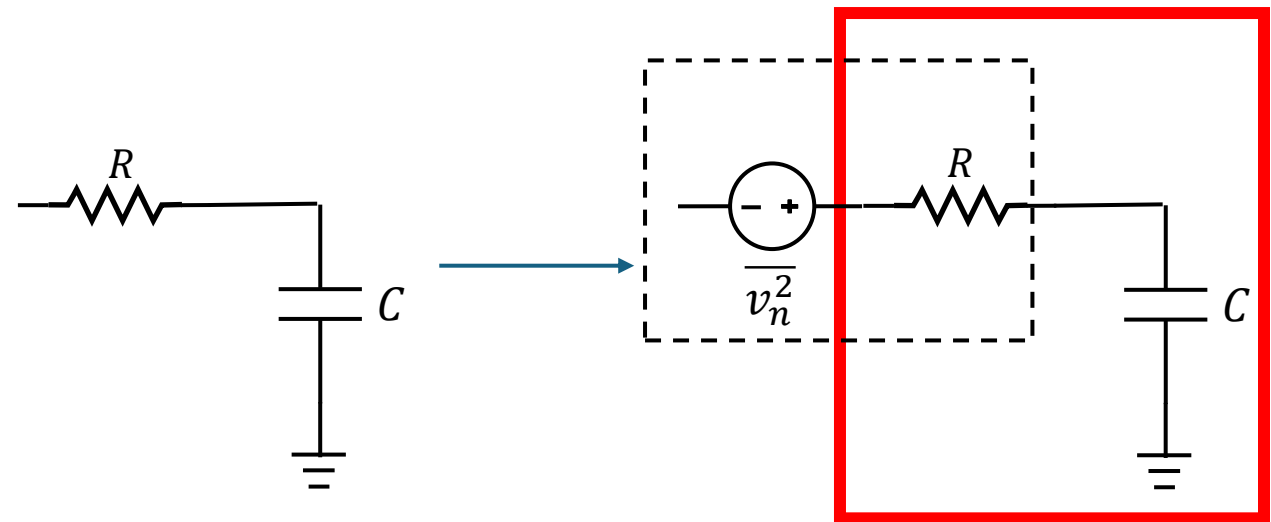
Thermal Noise - RC-low-pass-filter

- Calculate the effect of thermal noise on an RC-low pass filter.
 - What will be the noise RMS **voltage** at the output (across the capacitor)?



Thermal Noise - RC-low-pass-filter

- Calculate the effect of thermal noise on an RC-low pass filter.
 - What will be the noise RMS **voltage** at the output (across the capacitor)?
- Remind yourself:
 - An ideal capacitor does NOT generate any noise.
 - It is the equivalent series resistance in the capacitor that generates thermal noise.
 - We can lump this with the external resistor R if we like, but for now, let's consider the capacitor to be ideal.



Thermal Noise - RC-low-pass-filter

- Remind yourself:

- How do we calculate the output power spectral density when noise goes through a frequency dependent system?

$$S_{out}(f) = |H(j2\pi f)|^2 S_{in}(f)$$

- The noise source here is the resistor: generates white noise with power spectral density

$$S_{in}(f) = 4k_B T$$

- The output power is

$$P_{out} = \int_0^{\infty} |H(j2\pi f)|^2 \times 4k_B T df$$

- With the transfer function of the R-C filter $H(j2\pi f) = \frac{1}{1+j2\pi fRC}$

- Therefore,

$$P_{out} = 4k_B T \int_0^{\infty} \left| \frac{1}{1+j2\pi fRC} \right|^2 df$$

- But this is tedious!

Thermal Noise - RC-low-pass-filter

- Remind yourself:
 - If we had just a constant to integrate over, life would be much easier!
 - We know that we can get the **effective bandwidth** of this RC-filter by simply
 - considering it as an ideal-brick-wall low pass filter with cutoff frequency $f_{Eff} = \frac{\pi}{2} f_{-3}$ where $f_{-3} = \frac{1}{2\pi RC}$ is the -3dB cutoff frequency of the actual non-ideal filter!
 - And, for an ideal low pass filter, the power spectral density is just a constant until the cutoff frequency, and then zero!

$$H(s)_{Effective} = \begin{cases} 1 ; f < f_{Eff} \\ 0 ; f \geq f_{Eff} \end{cases}$$

- Therefore,

$$P_{out} = 4k_B T \int_0^{\infty} |H(s)_{Effective}|^2 df$$

$$\Rightarrow 4k_B T [f_{Eff} - 0]$$

- Therefore, we have, simply: $P_{out} = 4k_B T \Delta f_{Eff}$

Thermal Noise - RC-low-pass-filter

- Therefore, whenever you see an RC-network in a circuit, you can replace its collective noise with the noise of a brick-wall filter with an effective bandwidth of $\frac{\pi}{2} f_{-3}$

- This makes calculations much easier and we could directly arrive at the following formula, skipping all previous computations!

$$P_{out} = 4k_B T \Delta f_{eff}$$

- Plugging the values in:

$$P_{out} = 4k_B T \left[\frac{\pi}{2} f_{-3} - 0 \right] = 4k_B T \cdot \frac{\pi}{2} \cdot \frac{1}{2\pi RC} = \frac{k_B T}{RC}$$

- Therefore, the output noise **power** through an RC filter depends on the the temperature, the resistance, and the capacitance.
- If the capacitance is high, the noise power that gets through reduces.

Thermal Noise - RC-low-pass-filter

- What about the V_{RMS} ?
 - Since the entire power output was generated by just the resistor, we have;
 - $V_{RMS} = \sqrt{P_{out} \times R} = \sqrt{\frac{k_B T}{C}}$
 - Surprisingly, the R term vanished.
 - This means, the noise RMS **voltage** through a RC network does **NOT** depend on the resistance!
 - This is because, increasing R increases the noise voltage power spectral density, but at the same time, reduces the bandwidth by the same amount.

Shot Noise – Basics

- Observed whenever there is a potential barrier for charge to cross.
 - Ex: PN-junction.
- The charge either passes or not; discrete.
 - Compare to charge in a wire or resistor where it can go wherever it wants and it is a continuum.
 - But across a potential barrier, you know charge either stays on the two sides or crosses, no staying in between.

Shot Noise – Basics

Characteristics

- Characteristics:
 - Flicker noise is a current noise and its **current noise spectral density** (A^2/Hz units) is given as:
 - $S = 2qI_{DC}$ where $q = 1.6 \times 10^{-19}$ Coulomb
 - Modelled as a poisson process.
 - White noise – independent of absolute frequency, just like thermal noise.
 - Therefore, total noise current mean square $\overline{i_n^2} = 2qI_{DC}\Delta f$
 - i.e., because it is a flat PSD, you don't have to evaluate the integral, just multiply by bandwidth.
 - Independent of temperature, in contrast to thermal noise.
 - Therefore, even if you freeze a PN-junction to absolute zero, you will still have shot noise.
 - Increases with the DC current flowing through the junction.
 - Because every electron that reaches the junction will cross to the other side at different/random time intervals.
 - Therefore, the bias point current of a small signal circuit will determine the amount of shot noise.

Shot Noise – Basics

Significance

- Why we learn about thermal noise – Obvious – Resistances are everywhere.
 - Temperatures fluctuate => Thermal noise power changes.
 - Thermal noise current and voltage depends on the resistance value.
- But why do we learn about shot noise – Not obvious
 - Only becomes a problem when the current **variations** being measured are really low.
 - When DC current through a junction is high, the shot noise increases.
 - Then, isn't it counterintuitive that we worry about shot noise?
 - Shot noise increases when DC current is high, but when DC current itself is high, it will be much larger than the shot noise. So, why don't we just ignore shot noise?
 - We can, but only if the circuit was purely interested in the DC current!
 - Most circuits have a DC current bias around which small signals oscillate, and these small signals are what we are usually interested in.
 - The shot noise could be comparable to very small signals that vary on top of a large baseline current.
 - In these situations, shot noise may have to be considered.

Shot Noise – Basics

More about Shot Noise

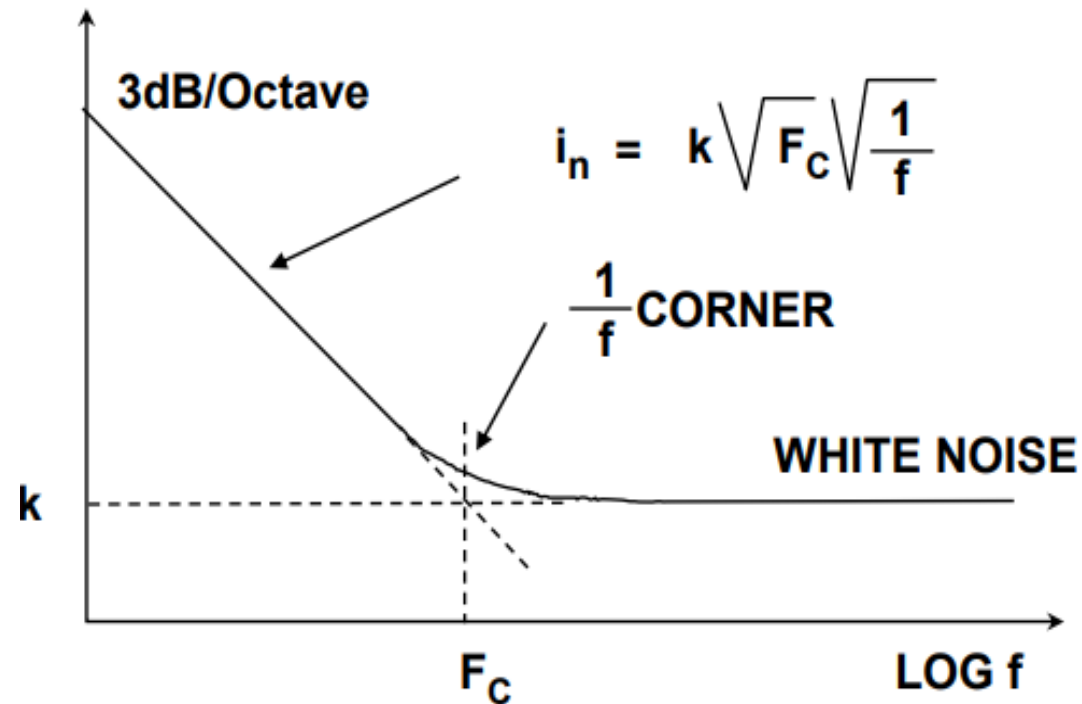
- In general however, shot noise is very small compared to thermal noise and flicker noise effects, especially in the low frequency, and high temperature domains.
- In **high frequency** and **low temperature** domain,
 - thermal noise will vanish due to low temp,
 - and flicker noise (which we will learn next) will vanish due to high frequency.
 - However, shot noise remains and will **dominate** as it is dependent neither on temperature nor on frequency.

1/f Noise – Basics

- Also called flicker noise, pink noise.
- Nobody knows exactly how this happens, but it is a real menace in semiconductors.
 - The hypothesis is:
 - There are energy trap states in materials, where charges can get randomly stuck for a while before being released.
 - The duration of staying ‘stuck’ is widely variable for each trap state.
 - However, when the effect of all these durations are superimposed, we get a $1/f$ spectrum.
- Resistors can also have flicker noise:
 - This is called ‘excess noise’ because it is in excess of the thermal noise.
 - However, this can be almost fully minimized with better materials and processing.
 - Carbon Composition resistors used to have a lot of excess noise.
 - Well-made metal-film resistors have very low excess noise.

1/f Noise – Basics

- Corner frequency for total noise is a figure of merit in devices.
 - The reason the corner exists is because of the 1/f noise.
 - The lower the corner frequency, the better.
 - Note: voltage noise and current noise both don't necessarily have the same corner frequency.



<https://www.analog.com/media/en/training-seminars/tutorials/MT-048.pdf>

1/f Noise – Basics

Characteristics

- Characteristics:

- Flicker noise is a current noise, and its **current noise spectral density** is given by:

- $S(f) = K_F I_{D_Q}^\alpha \cdot \frac{1}{f}$

- Here K_F and α are device dependent constants and have to be measured.
 - α is typically between 0.5 and 2, and is generally assumed to be close to 1.

- This is **NOT** white noise; i.e., the PSD is NOT flat.

- Therefore, you have to carry out the integration of the PSD to get the noise mean square current.

- $\overline{i_{n_F}^2} = \int_{f_L}^{f_H} K_F I_{D_Q}^\alpha \cdot \frac{1}{f} df = K_F I_{D_Q}^\alpha \ln\left(\frac{f_H}{f_L}\right)$

- Higher noise amplitude/power in low frequencies.
- At higher frequencies, the noise is minimal, and in this regime, white noise like shot and thermal noise dominates.

1/f Noise – Basics

Significance

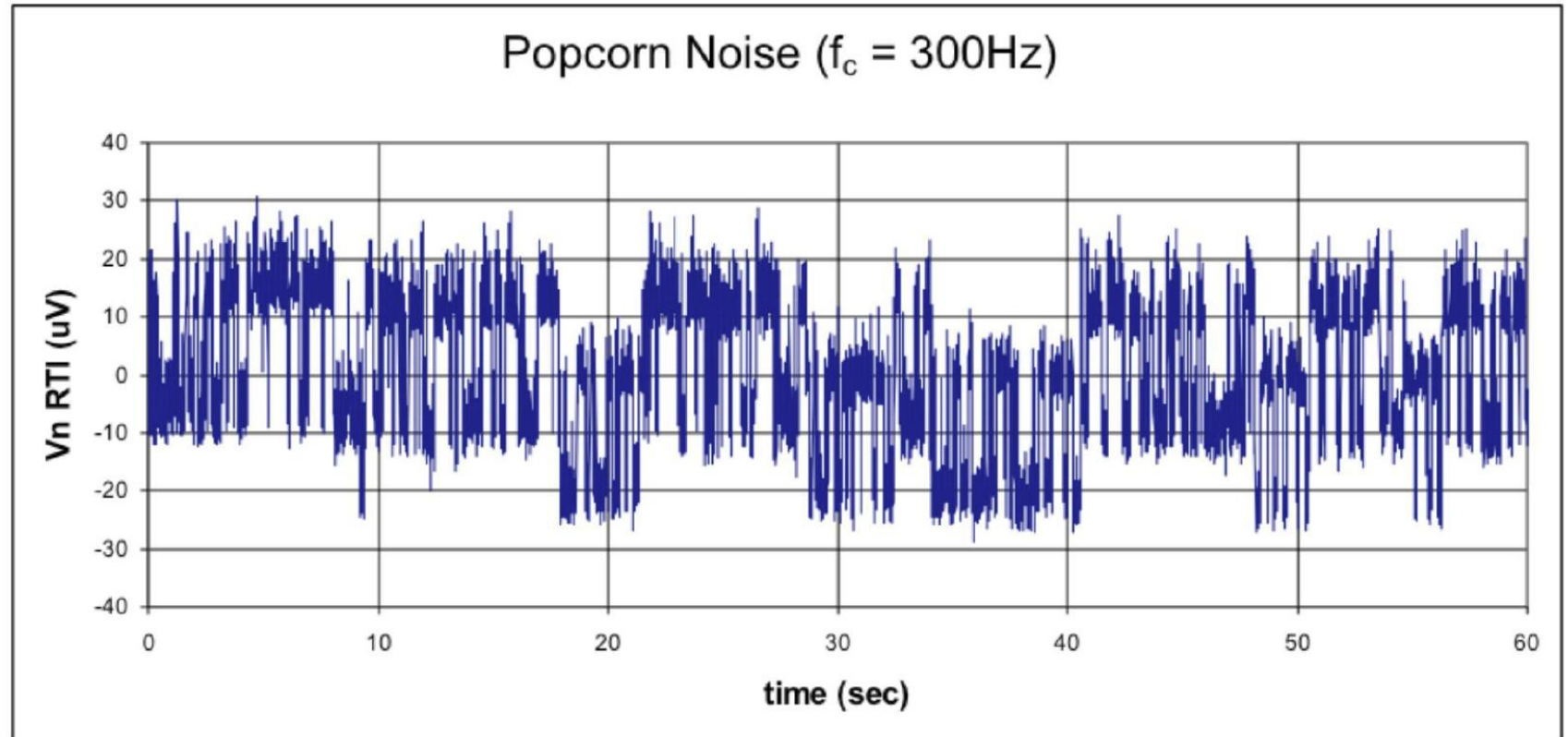
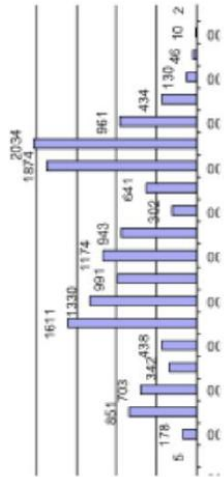
- Why do we learn about flicker noise?
 - In low frequency circuits where there are semiconductors involved, this noise dominates.
- It is higher in amplitude and power than thermal noise or shot noise and is the significant source of noise at these frequencies.
- Its contribution to total noise is characterized by the corner frequency f_c .
 - The low-frequency region dominated by flicker noise and the higher-frequency region dominated by the flat spectrum of white noise.
 - Mosfets have a high f_c (can be in the GHz range).
 - JFETs and BJTs have a lower f_c around 1 kHz.
 - JFETs however usually exhibit more flicker noise at low frequencies than BJTs
 - Can have f_c as high as several kHz in JFETs not selected for flicker noise.
- Therefore, a BJT would be better suited for an audio amplifier which operates in the kHz range.

Burst noise

- Also called popcorn noise.
- Creates sudden step-like transitions between two or more discrete voltage or current levels.
 - Every 20+ ms or so, the offset voltage randomly fluctuates in step like changes.
- Caused by imperfections in materials, and sudden trapping/detrapping of charge carriers in trap states.
 - Trap states are created due to contamination and lattice dislocations at surface which results from inappropriate processing techniques or poor raw material.
- This is not that visible nowadays due to better semiconductor manufacturing and processing techniques

Burst noise

- Observe the step-like transitions in the noise baseline.



Reference :Texas Instruments