



University of Moratuwa
Wisdom is all wealth

Noise and Interference in Instrumentation Part 3

Anton F. P. Van Putte, Electronic measurement systems, Taylor & Francis, 2nd Edition
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The Diode Noise Model

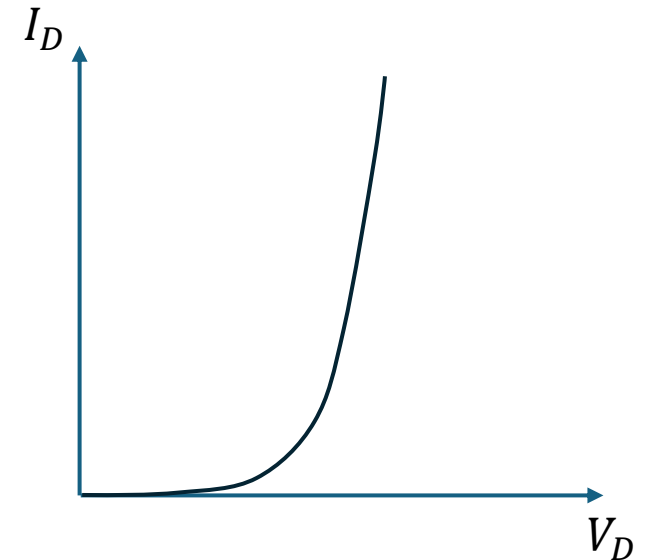
Modelling Noise: The Diode

The Diode Equation

- A diode is a non-linear device.
 - Described by the Shockley Diode Equation:

- $I_D = I_S \left(e^{\frac{V_D}{\eta V_T}} - 1 \right)$ where $V_T = \frac{k_B T}{q}$

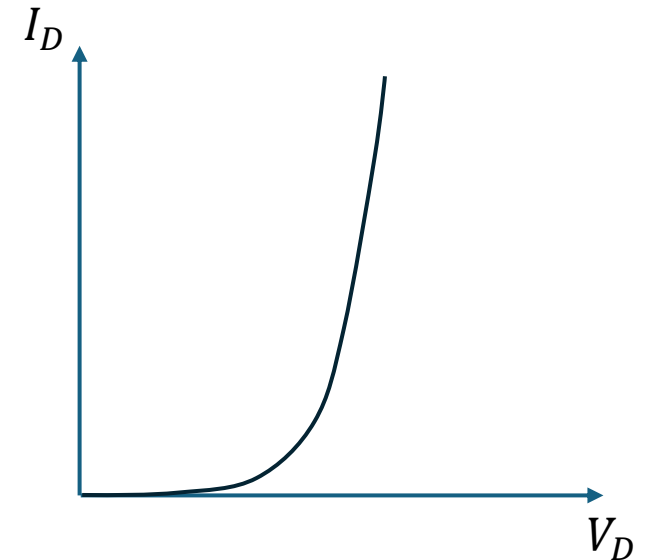
- I_S : Reverse Saturation Current (Order of 10^{-10} A)
 - I_D : Current through diode at steady-state
 - V_D : Voltage across diode at steady-state
 - η : Ideality Factor (Emission Coefficient) (Typically 1-2)
 - V_T : Thermal Voltage (≈ 25.7 mV at Room Temperature $T=298$ K)
 - T : Absolute temperature in Kelvin
 - q : Elementary Charge (1.602×10^{-19} C)
- Note: The Shockley equation assumes steady state:
 - i.e., it doesn't take into account how the diode responds when signals vary in time.



Modelling Noise: The Diode

The Diode Small Signal Model

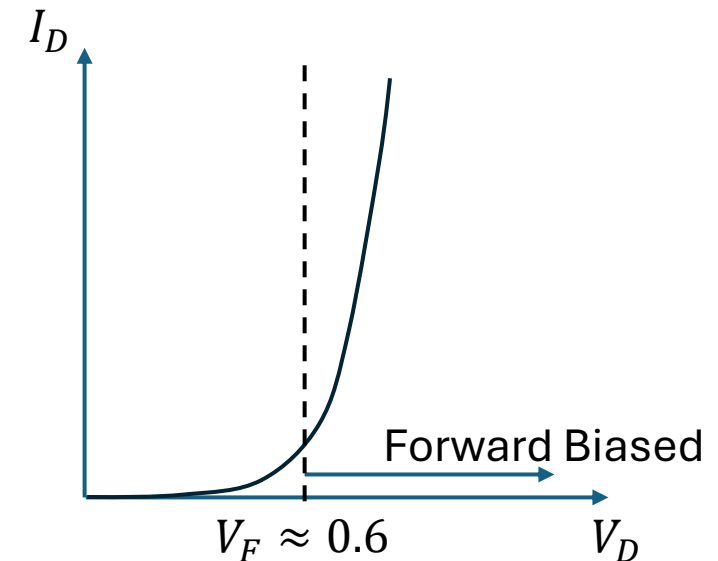
- Given this non-linearity, how do we simplify calculations?
 - Small signal approximation! => Linear Model
 - Why do we study about small signals?
 - Because they are more susceptible to noise.



Modelling Noise: The Diode

The Diode Small Signal Model

- The small signal approximation:
 - For a small signal scenario, we will usually have the diode forward biased.



Modelling Noise: The Diode

The Diode Small Signal Model

- The small signal approximation:
 - For a small signal scenario, we will usually have the diode forward biased.
 - If the signal only oscillates very little about the operating point Q , we can approximate the diode's characteristic curve in this small region to be linear.

- Slope of the diode curve: $\frac{dI_D}{dV_D} = \frac{d}{dV_D} I_S \left(e^{\frac{V_D}{\eta V_T}} - 1 \right) = \frac{I_S \times e^{\frac{V_D}{\eta V_T}}}{\eta V_T}$

- Substituting diode equation:

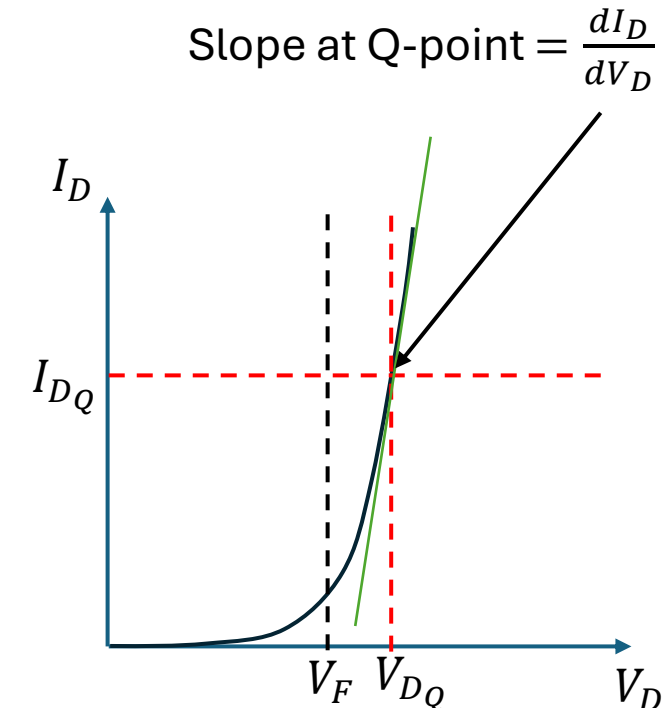
- $\frac{dI_D}{dV_D} = \frac{I_D + I_S}{\eta V_T}$

- Slope at the Q-point:

- $\frac{dI_D}{dV_D} \Big|_{I_D=I_{DQ}, V_D=V_{DQ}} = \frac{I_{DQ} + I_S}{\eta V_T} \approx \frac{I_{DQ}}{\eta V_T}$

- Since I_S is in the order of 10^{-10} A and I_D has already reached mA levels during forward bias, we have:

$$\frac{dI_D}{dV_D} \Big|_{I_D=I_{DQ}, V_D=V_{DQ}} \approx \frac{I_{DQ}}{\eta V_T}$$



Modelling Noise: The Diode

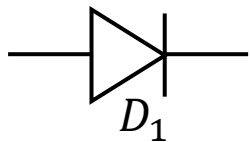
The Diode Small Signal Model

- The small signal approximation:

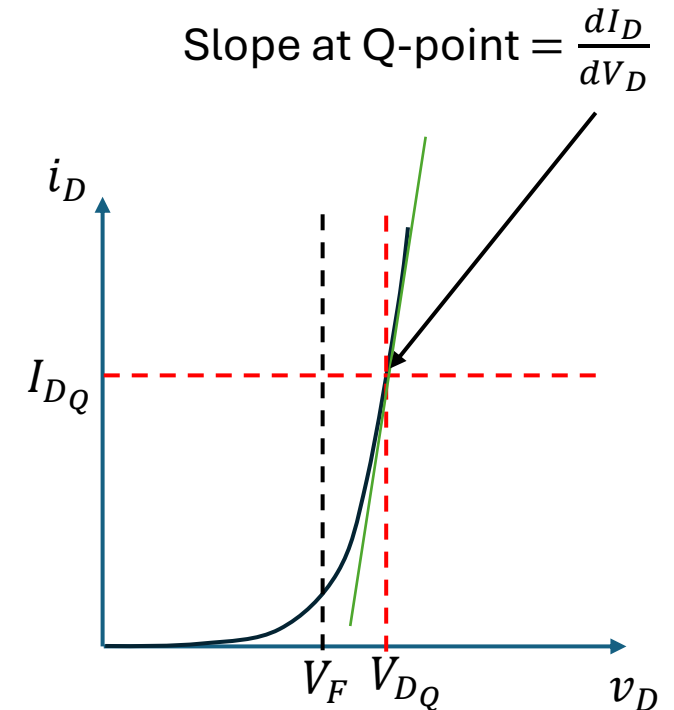
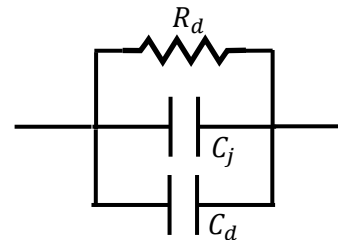
- Since $\frac{dI_D}{dV_D}$ has units of conductance, its inverse has units of resistance.
 - Therefore, for small signals, a diode in forward bias follows the same current-vs-voltage relationship that a resistor would do.
 - Therefore, we can model the diode as a simple resistor with resistance:

$$r_d = \frac{\eta V_T}{I_{DQ}}$$

- Here, r_d is called the dynamic resistance
- In addition, we have a junction capacitance (C_j) and diffusion capacitance (C_d) in parallel with r_d .
 - These take into account the response of the diode to time-varying signals.



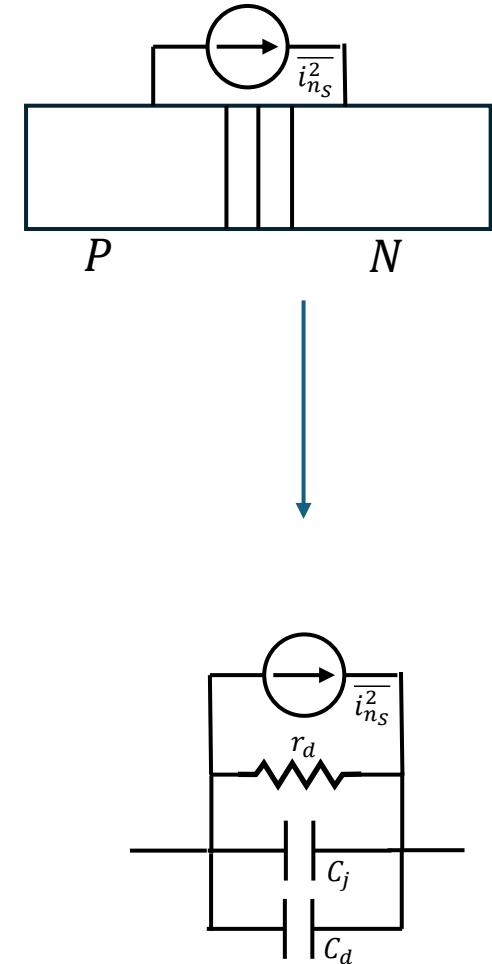
Small signal Approximation



Modelling Noise: The Diode

The Diode Noise Model

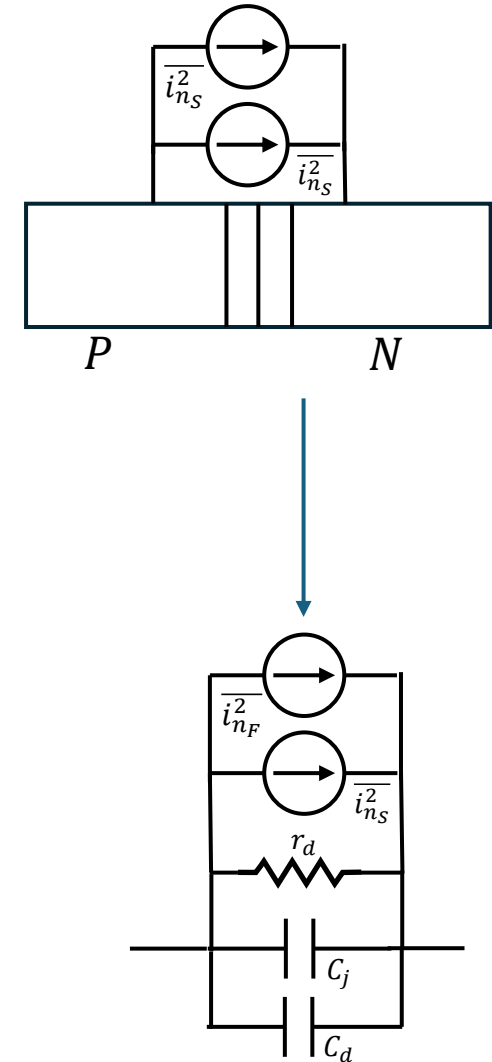
- A diode has three main sources of noise.
 - It has a PN-junction => Gives rise to shot noise.
 - Can be represented as a parallel current noise $\overline{i_{n_s}^2}$ across the junction, hence across r_d .
 - $\overline{i_{n_s}^2} = 2qI_{D_Q}\Delta f$
 - Here I_{D_Q} is the quiescent current.



Modelling Noise: The Diode

The Diode Noise Model

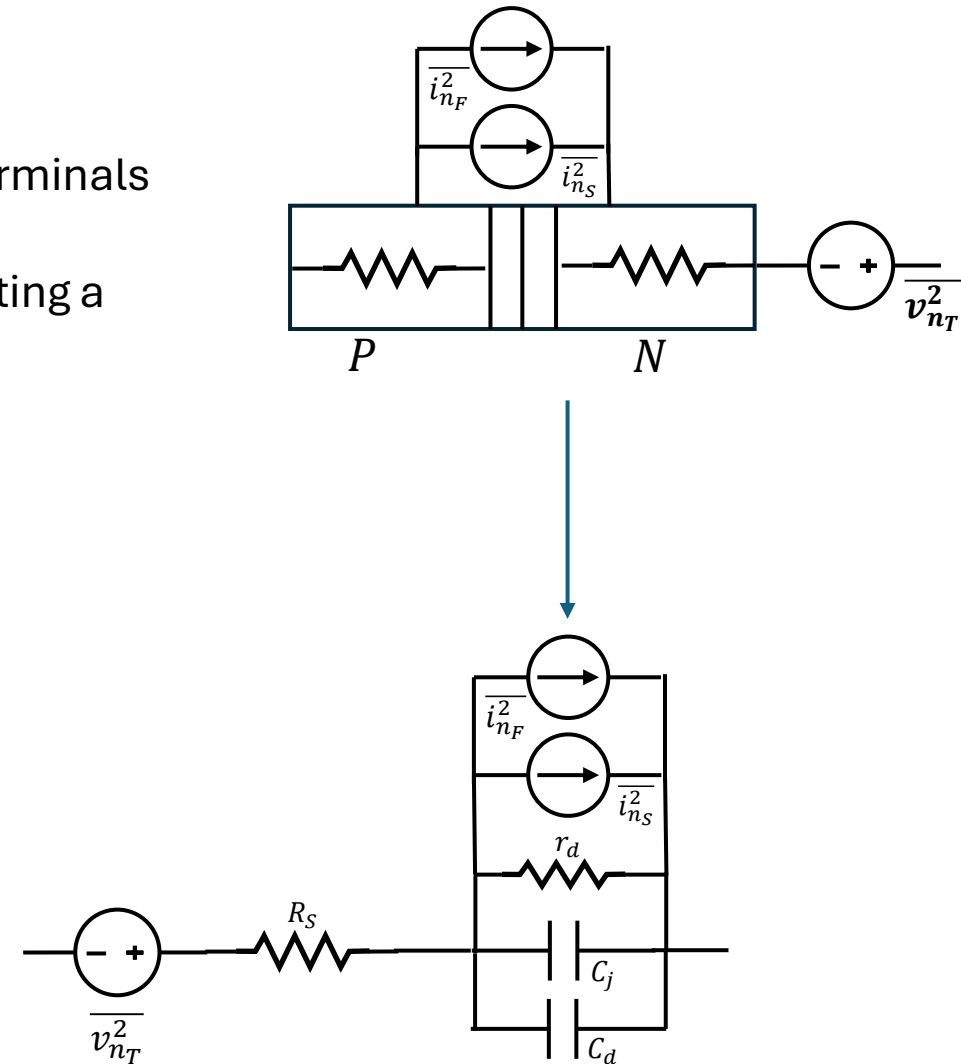
- A diode has three main sources of noise.
 - It is a semiconductor device => Has flicker noise.
 - Can also be represented as a parallel current noise $\overline{i_{n_F}^2}$ across r_d .
 - $\overline{i_{n_F}^2} = \int_{f_L}^{f_H} K_F I_{DQ}^\alpha \cdot \frac{1}{f} df = K_F I_{DQ}^\alpha \ln\left(\frac{f_H}{f_L}\right)$
 - where K_F and α are device dependent constants.



Modelling Noise: The Diode

The Diode Noise Model

- A diode has three main sources of noise.
 - It has resistance => Thermal noise.
 - The resistance is mainly in the contacts between the external terminals and the silicon substrate, and within the substrate itself.
 - We can represent this with a resistor R_s in series with r_d , generating a voltage noise $\overline{v_{n_T}^2}$.
 - Why don't we add thermal noise to r_d ?
 - r_d is not a real resistor - It is a dynamic resistance that we added to resemble the I-V relationship of the diode.



Modelling Noise: The Diode

Total Noise

- Calculate the total **voltage** noise generated by a diode.
 - For simplicity, ignore C_d and C_j (Assume open circuit).
 - This is reasonable for frequencies up to the MHz range where the roll-off due to the capacitors have not yet begun :
 - We have the thermal, flicker, and shot noise sources.
 - We cannot add these three sources blindly as they sit in different branches: R_s 's thermal noise is a series voltage source, and shot + flicker noises are current sources in parallel with r_d .
 - Fix: Convert the parallel current sources into an equivalent series (Thevenin) voltage across r_d :
 - $\overline{v_{r_d}^2} = (\overline{i_{n_S}^2} + \overline{i_{n_F}^2}) \times r_d^2$
 - Now everything is in series, hence with the assumptions that noise has zero mean, are uncorrelated, and the fact that KVL is applicable:
 - $\overline{v_{n_{Tot}}^2} = \overline{v_{n_T}^2} + \overline{v_{r_d}^2} = 4k_B T \Delta f R_s + (\overline{i_{n_S}^2} + \overline{i_{n_F}^2}) \times r_d^2$
 - Let the bandwidth Δf be between f_L and f_H .
 - $\overline{v_{n_{Tot}}^2} = 4k_B T (f_H - f_L) R_s + \left(2q I_{D_Q} (f_H - f_L) + K_F I_{D_Q}^\alpha \ln \left(\frac{f_H}{f_L} \right) \right) \times r_d^2$

Modelling Noise: The Diode

Total Noise

- Calculate the total **voltage** noise generated by a diode.

- $\overline{v_{n_{Tot}}^2} = 4k_B T (f_H - f_L) R_s + 2q I_{D_Q} r_d^2 (f_H - f_L) + K_F I_{D_Q}^\alpha \ln\left(\frac{f_H}{f_L}\right) r_d^2$

- Since $I_{D_Q} = \frac{\eta V_T}{r_d}$ and $V_T = \frac{k_B T}{q}$

- $\overline{v_{n_{Tot}}^2} = 4k_B T (f_H - f_L) R_s + \frac{2q \times \eta k_B T}{q \times r_d} r_d^2 (f_H - f_L) + K_F I_{D_Q}^\alpha \ln\left(\frac{f_H}{f_L}\right) r_d^2$

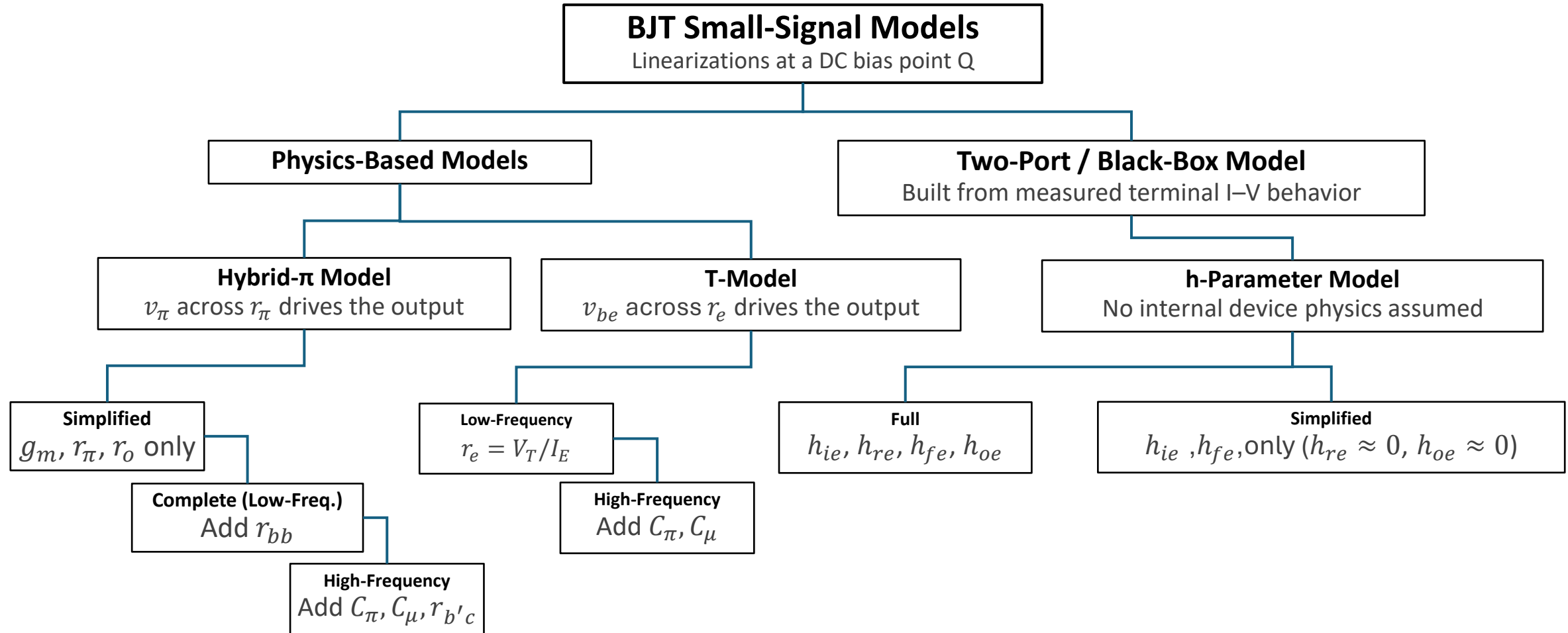
- $\therefore \overline{v_{n_{Tot}}^2} = 2k_B T (f_H - f_L) (2R_s + \eta r_d) + K_F I_{D_Q}^\alpha \ln\left(\frac{f_H}{f_L}\right) r_d^2$

The BJT Noise Model

Modelling Noise: The BJT

The BJT Small Signal Model

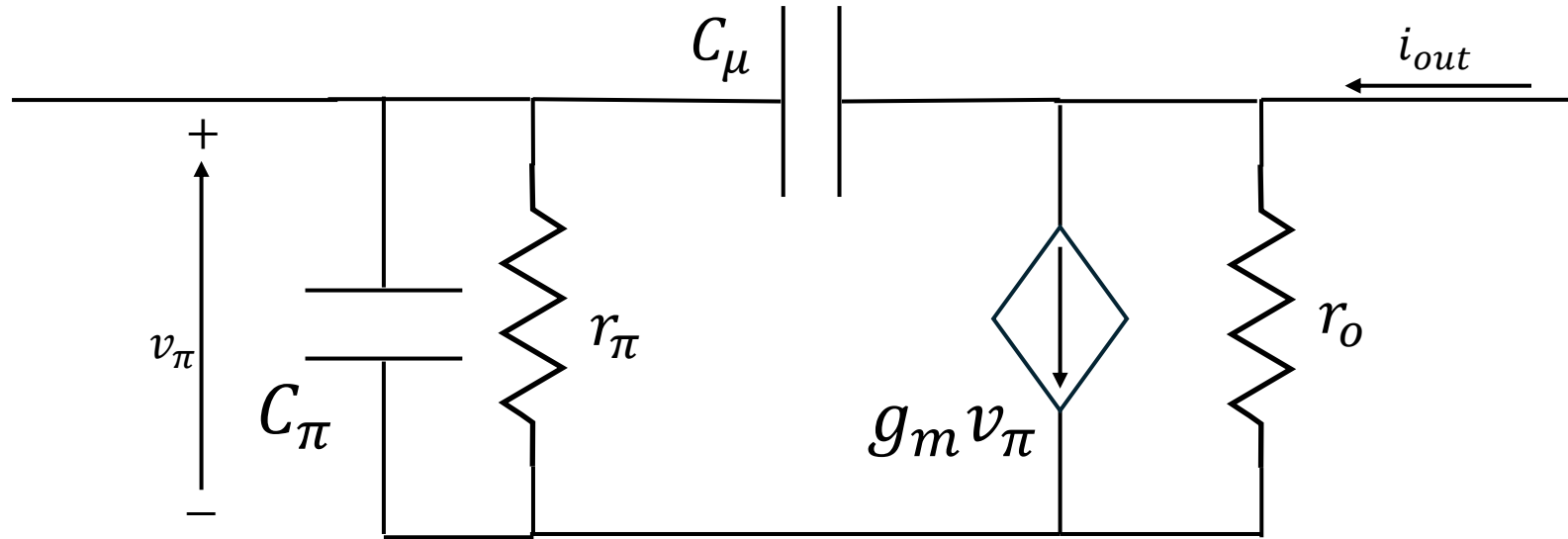
- There are several small signal models for a BJT



Modelling Noise: The BJT

The BJT Small Signal Model

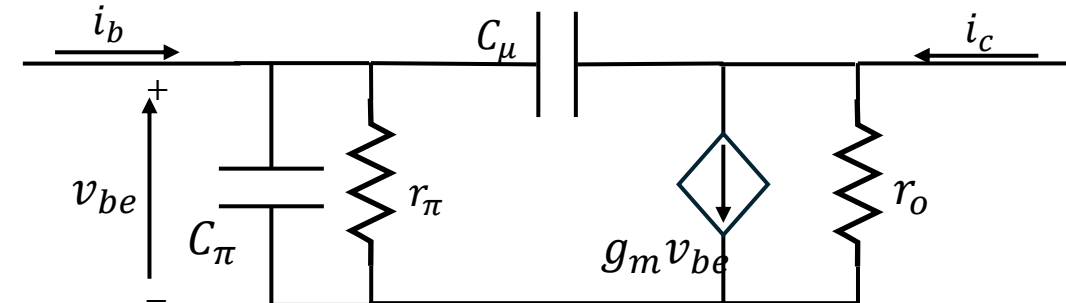
- Let's use the high frequency π model:



Modelling Noise: The BJT

DC Current gain calculation

- Calculate the DC and AC current gains β_{DC} and β_{AC} respectively.
- From linearizing the transistor I_C vs V_{BE} relation (with V_{CE} shorted) ($I_C \approx I_S e^{\frac{V_{BE}}{V_T}}$) around operating point Q, we obtain for small signals:
 - $i_c = g_m v_{be}$ where $g_m = \frac{I_{CQ}}{V_T}$ where $V_T = \frac{k_B T}{q}$
 - Note: The full form of i_c is $i_c = g_m v_{be} + \frac{v_{ce}}{r_o}$.
 - However, the effect of r_o only arises when a load is connected at the collector.
 - Therefore, to characterize just the transistor, we can short circuit the output.
 - Also, $i_b = \frac{v_{be}}{r_\pi}$
 - Therefore, the DC current gain $\beta_{DC} = \frac{I_C}{I_B} = g_m r_\pi$



Modelling Noise: The BJT

AC Current gain calculation

- Calculate the DC and AC current gains β_{DC} and β_{AC} respectively.
 - Next, we calculate the AC current gain β_{AC} :
 - For AC small signals, the capacitors must be considered.
 - With the C-E shorted, we have the total impedance between B-E (z_{be}) as:

$$z_{be} = r_{\pi} // z_{C_{\pi}} // z_{C_{\mu}} \Rightarrow r_{\pi} // \left(\frac{1}{j\omega(C_{\pi} + C_{\mu})} \right) = \frac{r_{\pi}}{1 + j\omega r_{\pi}(C_{\pi} + C_{\mu})}$$

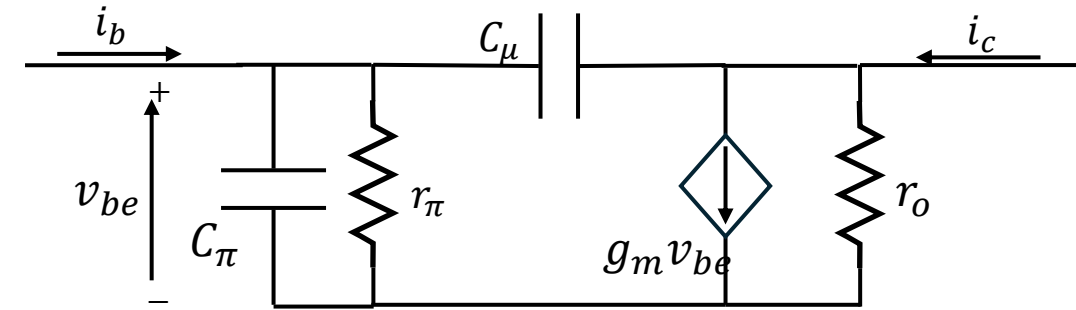
$$\therefore v_{be} = z_{be} \times i_b = i_b \frac{r_{\pi}}{1 + j\omega r_{\pi}(C_{\pi} + C_{\mu})}$$

$$\text{• Since } v_{be} = \frac{i_c}{g_m}, \text{ we have } \beta_{AC} = \frac{i_c}{i_b} = \frac{g_m r_{\pi}}{1 + j\omega r_{\pi}(C_{\pi} + C_{\mu})}$$

$$\text{• Since } \beta_{DC} = g_m r_{\pi}, \text{ we have:}$$

$$\beta_{AC} = \frac{\beta_{DC}}{1 + j\omega r_{\pi}(C_{\pi} + C_{\mu})}$$

- Note: This formula looks very similar to the transfer function of a R-C low pass filter with $f_{-3} = \frac{1}{2\pi r_{\pi}(C_{\pi} + C_{\mu})}$ albeit with a gain of β_{DC} .



Modelling Noise: The BJT

Usable frequency range for a BJT

- We can re-write the above as:

- $$\beta_{AC} = \frac{\beta_{DC}}{1 + j\omega g_m r_\pi \left(\frac{C_\pi + C_\mu}{g_m} \right)}$$

- Since $\beta_{DC} = g_m r_\pi$, we have

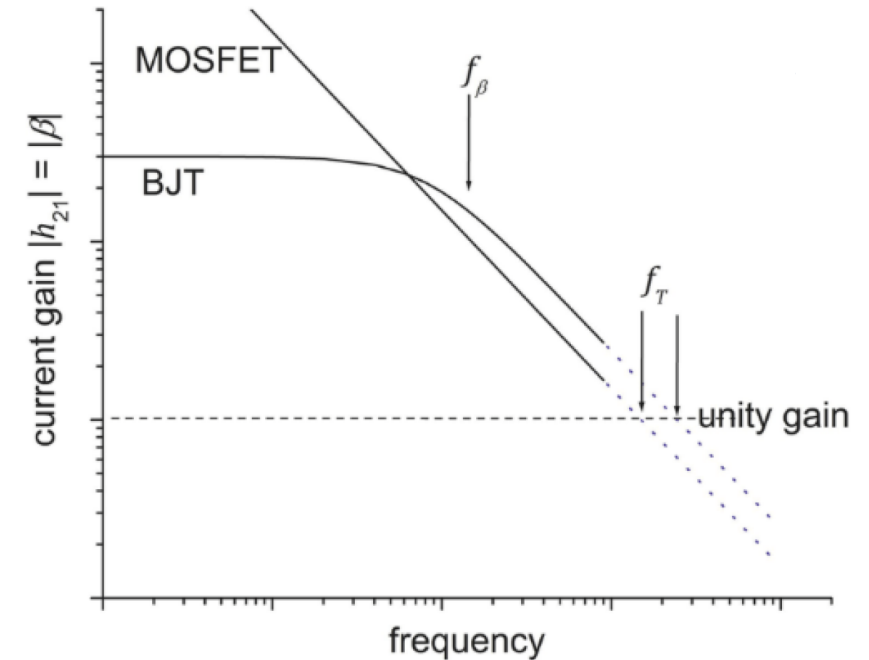
- $$\beta_{AC} = \frac{\beta_{DC}}{1 + j\omega \beta_{DC} \left(\frac{C_\pi + C_\mu}{g_m} \right)} = \frac{1}{\frac{1}{\beta_{DC}} + j\omega \left(\frac{C_\pi + C_\mu}{g_m} \right)}$$

- Define $\omega_T = \frac{g_m}{(C_\pi + C_\mu)}$

- Then, $\beta_{AC} = \frac{1}{\frac{1}{\beta_{DC}} + j\left(\frac{\omega}{\omega_T}\right)}$

- Squared magnitude of β_{AC} :

- $$|\beta_{AC}|^2 = \beta_{AC} \times \beta_{AC}^* = \frac{1}{\frac{1}{\beta_{DC}} + j\left(\frac{\omega}{\omega_T}\right)} \times \frac{1}{\frac{1}{\beta_{DC}} - j\left(\frac{\omega}{\omega_T}\right)} = \frac{1}{\frac{1}{\beta_{DC}^2} + \left(\frac{\omega}{\omega_T}\right)^2}$$



Modelling Noise: The BJT

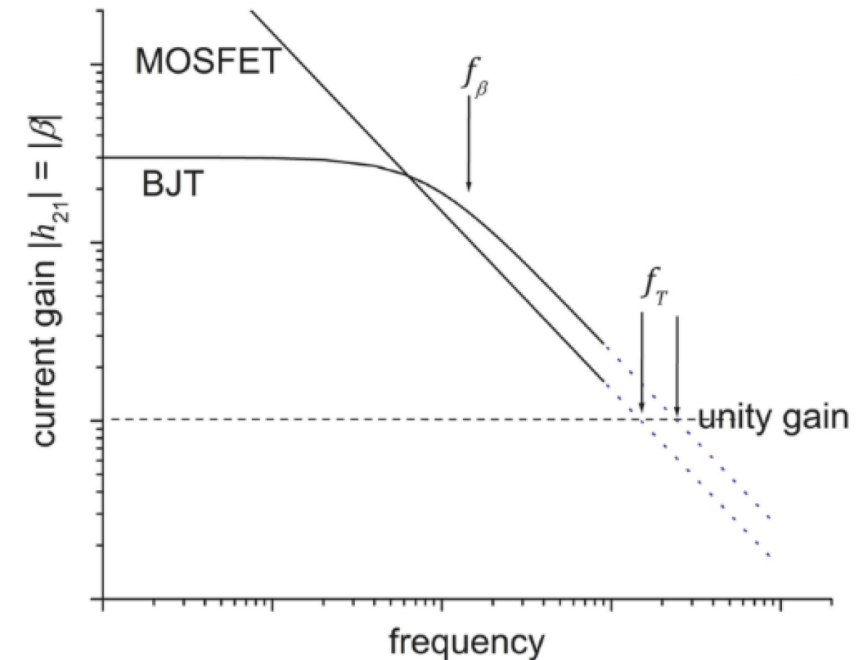
Usable frequency range for a BJT

- $\therefore |\beta_{AC}|^2 = \frac{1}{\frac{1}{\beta_{DC}^2} + \left(\frac{\omega}{\omega_T}\right)^2}$ with $\omega_T = \frac{g_m}{(C_\pi + C_\mu)}$

- When $\omega \rightarrow \omega_T$; $\frac{\omega}{\omega_T} \rightarrow 1$

$$\therefore |\beta_{AC}|^2 \rightarrow \frac{1}{1 + \frac{1}{\beta_{DC}^2}} \approx 1 \quad (\because \beta_{DC} \gg 1)$$

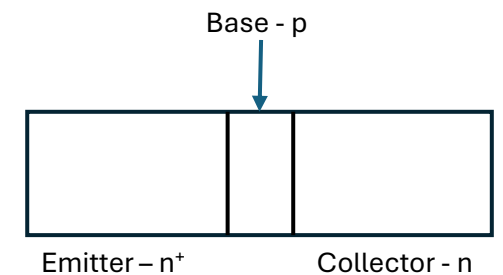
- i.e., as ω approaches ω_T , the current gain drops to 1.
- Therefore, ω_T is called the unity gain frequency.
- Beyond that, the BJT no longer works as an amplifier.
- Since $g_m = \frac{I_{CQ}}{V_T}$, the unity gain frequency is **not** merely a static characteristic of the BJT, but changes with the quiescent current.
 - $\therefore \omega_T$ is the maximum operating frequency for a BJT for a given quiescent collector current I_{CQ} .



Modelling Noise: The BJT

Noise Sources in a BJT

- There are 6 main noise sources in a BJT.
 - Three of them are thermal noise sources.
 - There are two main shot noise sources.
 - There is a flicker noise source.



Modelling Noise: The BJT

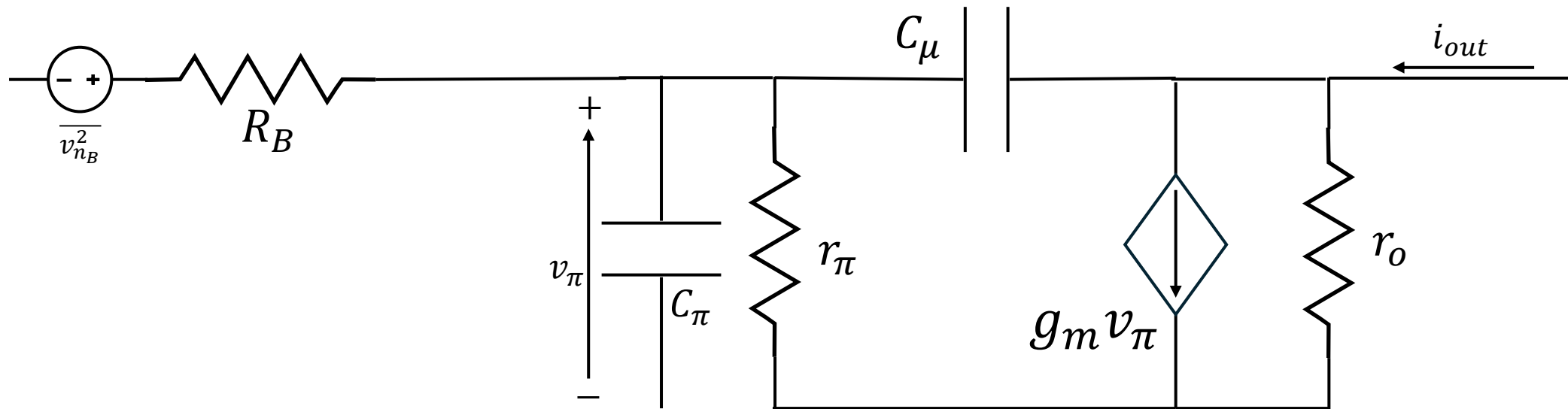
Noise Sources in a BJT

- Thermal noise sources in a BJT:
 - Thermal noise sources in the emitter, base, and collector regions due to the terminal and semiconductor substrate bulk resistances.
 - These are denoted as R_E , R_B , and R_C and may be added to the small signal model in series with the emitter, base, and collector terminals respectively
 - The resistances R_E and R_C are generally ignored because they are small.
 - The emitter is highly doped, hence minimal resistance.
 - The collector is lightly doped at the C-B junction but has a wide cross section and a well doped substrate for carrying signals outward, hence having low resistance.
 - The base resistance however cannot be ignored due to the following reasons:
 - The base is a very narrow region => comparatively high resistance => High noise voltage.
 - Only moderately doped => comparatively high resistance => High noise voltage.
 - Noise at the base will be amplified at the output due to transistor β being high.
- Note: The dynamic resistances r_π and r_o do not generate noise as they are not real resistors (explained in the diode model).

Modelling Noise: The BJT

Noise Sources in a BJT

- Thermal noise sources in a BJT:



Modelling Noise: The BJT

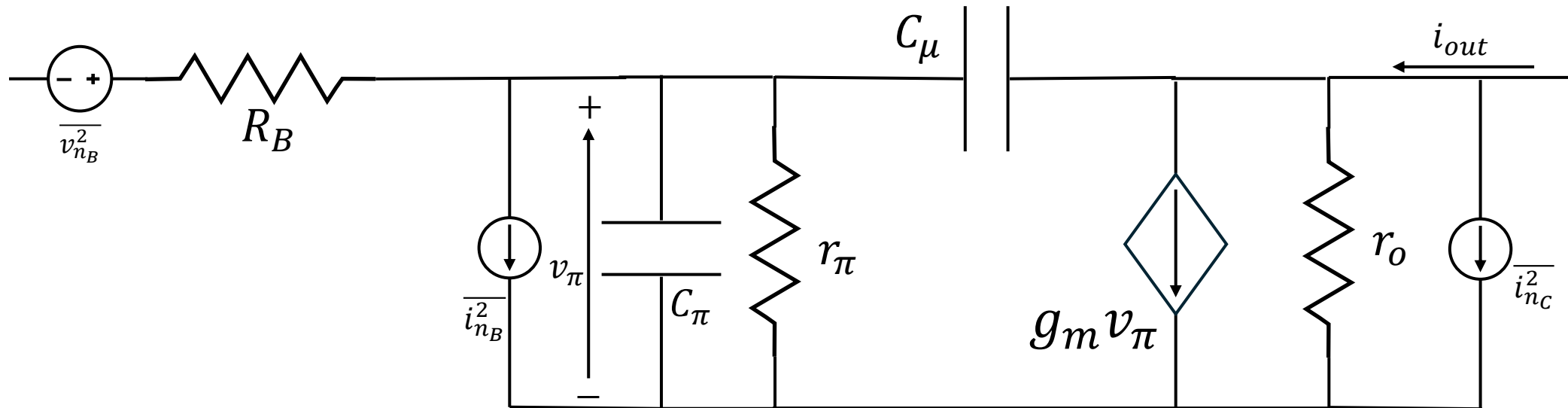
Noise Sources in a BJT

- Shot noise sources in a BJT.
 - There are two paths for current in a BJT, both that have to cross potential barriers => Shot noise!
 - When BJT operates, the base voltage V_{BE} causes the B-E junction to forward bias.
 - Therefore, electrons rush from the highly doped emitter to the base region.
 - However, since the base is very narrow and only moderately doped, these excess electrons only meet a little number of free holes, hence quickly diffuse through the base region and are accelerated into the collector region.
 - Therefore, the stream of electrons injected into the base from the emitter I_E is almost equal to the amount of electrons exiting the base into the collector region I_C : i.e., $I_C \approx I_E$.
 - Therefore, electrons moving from the emitter to the base and then to the collector can be considered as a single motion through a potential barrier, and is modeled as a shot noise source attributed to the collector current.
 - This is called **collector current shot noise** => $\overline{i_{n_C}^2} = 2qI_{C_Q}\Delta f$ and is modelled in parallel with the C-E junction (output) where I_{C_Q} is the **DC** collector current at the quiescent point.

Modelling Noise: The BJT

Noise Sources in a BJT

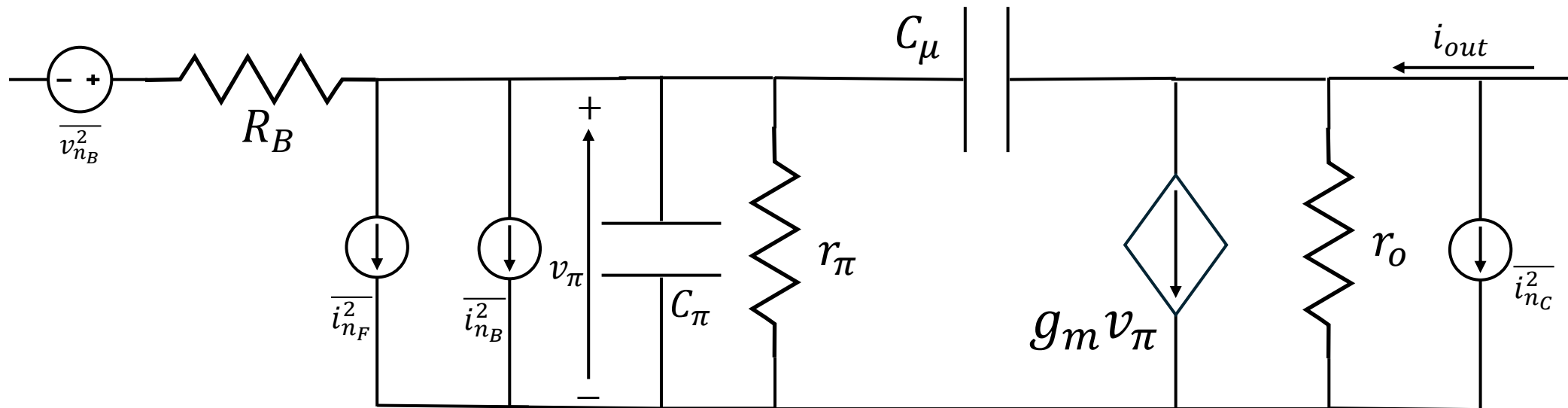
- Shot noise sources in a BJT.
 - The difference between I_E and I_C is due to the removal of the few charges that recombined in the base region, and this is equal to the base current I_B .
 - Therefore, I_B has to cross the base-emitter PN junction, hence will generate shot noise.
 - This is called **base current shot noise** $\Rightarrow \overline{i_{n_B}^2} = 2qI_{B_Q}\Delta f$ and is modelled in parallel with the B-E junction (input) where I_{B_Q} is the **DC** base current at the quiescent point.



Modelling Noise: The BJT

Noise Sources in a BJT

- Flicker noise sources in a BJT.
 - BJT's have trap states like any other semiconductor, hence flicker noise.
- Flicker noise is modelled as a current noise source $\overline{i_{n_F}^2}$ in parallel with the B-E junction (Input).
- $\overline{i_{n_F}^2} = KI_{BQ}^\alpha \ln\left(\frac{f_H}{f_L}\right)$ where $f_H - f_L = \Delta f$ is the bandwidth



Modelling Noise: The BJT

Input Referred Noise Calculation

- Calculate the input referred noise model for the BJT
 - Since the BJT is a 2-port device, the equivalent noise model requires a series voltage noise source **AND** a parallel current noise source across the input port.
 - We want to have all the sources transferred to the input side.
 - Base bulk resistance thermal noise, the base recombination current shot noise, and the flicker noise are already on the input port.
 - Only collector current shot noise $\overline{i_{n_C}^2}$ remains on the output side.
 - Let's refer any input referred quantity with a ' $-ir$ ' subscript.
 - Ex: If the collector shot current is $\overline{i_{n_C}^2}$, its input referred current noise portion is $\overline{i_{n_C-ir}^2}$

Modelling Noise: The BJT

Input Referred Noise Calculation

- Calculate the input referred noise model for the BJT
 - Collector current shot noise referred to the input:
 - We can refer its contribution at the input as an equivalent voltage noise source **and** an equivalent current noise source.
 - To calculate the input-referred **current** noise contribution $\overline{i_{n_c-ir}^2}$ of the collector shot noise $\overline{i_{n_c}^2}$, we divide the collector current shot noise by the squared magnitude of the **AC** current gain $|\beta_{AC}|^2$.
 - Note: β_{AC} is a complex number because it has phase information.
 - Therefore, it must be converted to a magnitude before doing noise calculations.
 - $$|\beta_{AC}|^2 = \frac{\beta_{DC}^2}{1 + (\omega r_\pi (C_\pi + C_\mu))^2} \Rightarrow \therefore \overline{i_{n_c-ir}^2} = \frac{\overline{i_{n_c}^2}}{|\beta_{AC}|^2} = 2qI_{CQ}\Delta f \times \frac{1 + (\omega r_\pi (C_\pi + C_\mu))^2}{\beta_{DC}^2}$$
 - Since $I_{CQ} = I_{BQ} \times \beta_{DC}$, and $\beta_{DC} = g_m r_\pi$ we have $\frac{I_{CQ}}{\beta_{DC}^2} = \frac{I_{BQ}}{\beta_{DC}} = \frac{I_{BQ}}{g_m r_\pi}$
 - $$\therefore \overline{i_{n_c-ir}^2} = 2q\Delta f I_{BQ} \left(\frac{1 + (\omega r_\pi (C_\pi + C_\mu))^2}{g_m r_\pi} \right)$$

Modelling Noise: The BJT

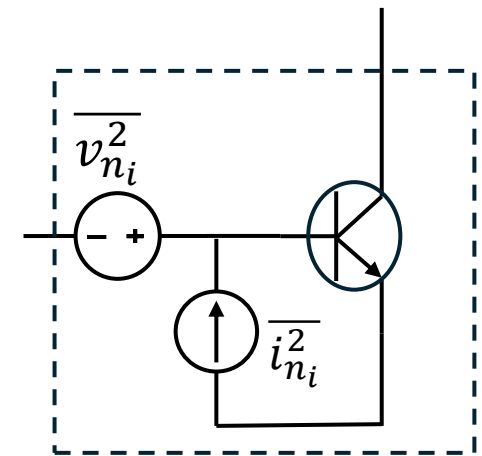
Input Referred Noise Calculation

- Calculate the input referred noise model for the BJT
 - Collector current shot noise referred to the input:
 - To calculate the input-referred **voltage** noise contribution $\overline{v_{n_c-ir}^2}$ of the collector shot noise $\overline{i_{n_c}^2}$, we divide collector current shot noise by the squared **transconductance** g_m (Already an AC quantity).
 - $\therefore \overline{v_{n_c-ir}^2} = \frac{\overline{i_{n_c}^2}}{g_m^2} = \frac{2qI_{CQ}\Delta f}{g_m^2}$
 - Since $I_{CQ} = I_{BQ} \times \beta_{DC}$, and $\beta_{DC} = g_m r_\pi$ we have $\frac{I_{CQ}}{g_m^2} = I_{BQ} \frac{r_\pi}{g_m}$
 - $\therefore \overline{v_{n_c-ir}^2} = 2qI_{BQ}\Delta f \frac{r_\pi}{g_m}$

Modelling Noise: The BJT

Input Referred Noise Model

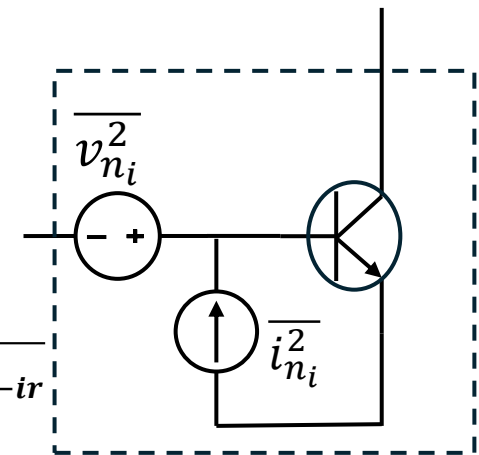
- Calculate the input referred noise model for the BJT
 - Therefore, the equivalent input referred noise model for the BJT:
 - Current noise:
 - $\overline{i_{n_i}^2} = \overline{i_{n_B}^2} + \overline{i_{n_{C-ir}}^2} + \overline{i_{n_F}^2}$
 - The above addition is possible only when noise sources have zero mean, are uncorrelated, and KVL/KCL permits the addition.
 - In reality, the collector noise and base noise are somewhat correlated. However, for the purpose of this course, we consider them uncorrelated.
 - $\therefore \overline{i_{n_i}^2} = 2qI_{B_Q}\Delta f + 2q\Delta f I_{B_Q} \left(\frac{1 + (\omega r_\pi (C_\pi + C_\mu))^2}{g_m r_\pi} \right) + KI_{B_Q}^\alpha \ln \left(\frac{f_H}{f_L} \right)$
 - $\Rightarrow \therefore \overline{i_{n_i}^2} = 2q\Delta f I_{B_Q} \left(1 + \left(\frac{1 + (\omega r_\pi (C_\pi + C_\mu))^2}{g_m r_\pi} \right) \right) + KI_{B_Q}^\alpha \ln \left(\frac{f_H}{f_L} \right)$
 - Voltage noise:
 - $\overline{v_{n_i}^2} = \overline{v_{n_B}^2} + \overline{v_{n_{C-ir}}^2} = 4k_B T \Delta f r_b + 2qI_{B_Q} \Delta f \frac{r_\pi}{g_m}$
 - The above addition is possible only when noise sources have zero mean, are uncorrelated, and KVL/KCL permits the addition.



Modelling Noise: The BJT

Input Referred Noise Model

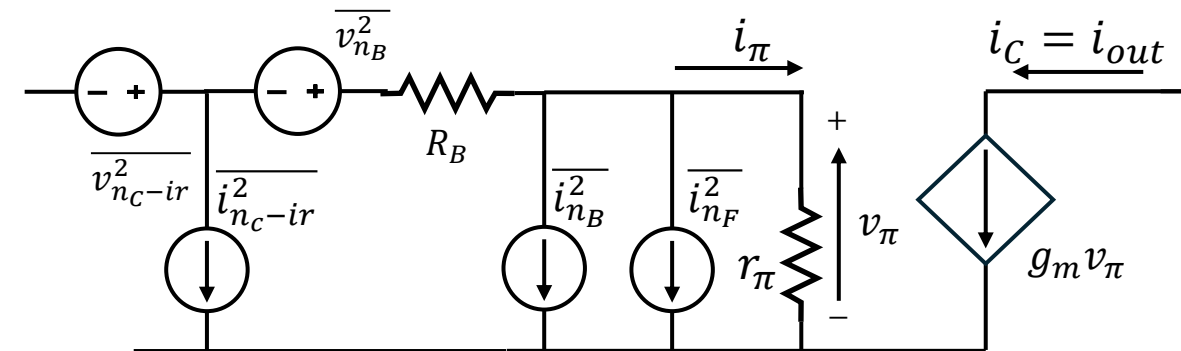
- Calculate the input referred noise model for the BJT
 - IMPORTANT:
 - The input referred terms of the collector shot current $\overline{i_{n_{C-ir}}^2}$ and $\overline{v_{n_{C-ir}}^2}$ are **NOT** independent!
 - They are two pieces representing the **same noise source**!
 - Therefore, when calculating total noise at the output, the noise contributions from **these two terms** can **NOT** simply be added as squared terms!
 - Ex:
 - Calculate the **total noise** at the output of a BJT amplifier operating in common emitter configuration (assume low frequency operation) - $|\beta_{AC}| \approx |\beta_{DC}|$
 - We know that
 - $\overline{i_{n_i}^2} = \overline{i_{n_B}^2} + \overline{i_{n_{C-ir}}^2} + \overline{i_{n_F}^2}$ and
 - $\overline{v_{n_i}^2} = \overline{v_{n_B}^2} + \overline{v_{n_{C-ir}}^2}$
 - In the above formulae, we never directly add $\overline{i_{n_{C-ir}}^2}$ with $\overline{v_{n_{C-ir}}^2}$ yet.
 - Therefore, no issues yet.
 - But, to calculate the total noise at the output, we need to combine $\overline{i_{n_{C-ir}}^2}$ with $\overline{v_{n_{C-ir}}^2}$



Modelling Noise: The BJT

Input Referred Noise Model

- Use the input referred noise model to calculate the total noise at the output of the BJT
 - Assuming low frequency operation, we can use a low frequency small signal model to calculate the total output noise. – Let's calculate the total output noise **current**.
 - We can use superposition principle to independently analyze each source.
 - Then, $\overline{i_{out}^2} = \overline{i_C^2} = g_m^2 \times \overline{v_\pi^2}$
 - Therefore, we need to find the voltage produced at r_π by each source.
 - Assume an ideal signal source (source impedance $R_S = 0$) is connected at the input.
 - Therefore, you can short circuit the input terminal to ground when doing superposition calculations.
 - If you want to be more rigorous, you can leave the R_S in place, and it will work in series with R_B and the $(R_B + R_S)$ will be parallel to r_π .

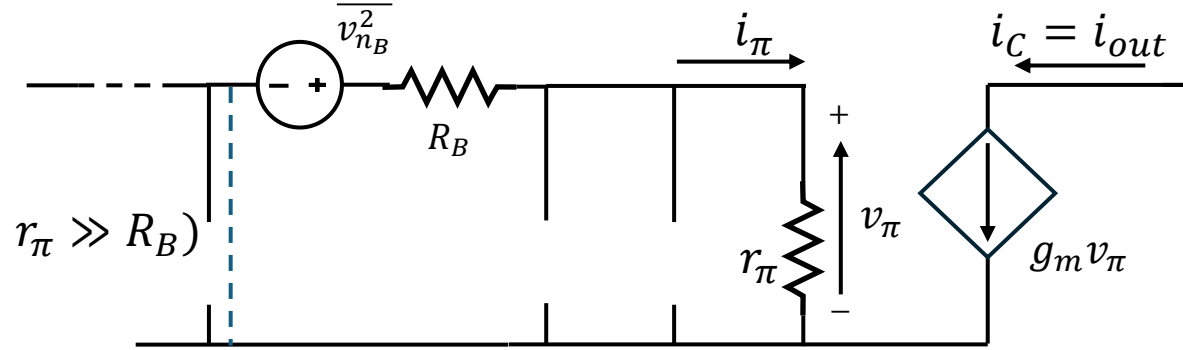


Modelling Noise: The BJT

Input Referred Noise Model

- Step 1: For base thermal noise:

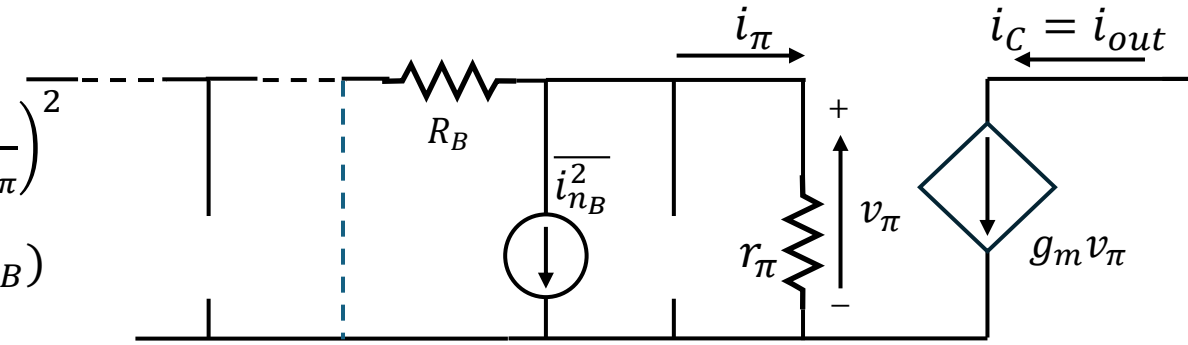
$$\overline{i_{out}^2}|_{n_{B_{Thermal}}} = g_m^2 \times \overline{v_{n_B}^2} \times \left(\frac{r_\pi}{R_B + r_\pi} \right)^2 \approx g_m^2 \times \overline{v_{n_B}^2} \quad (\because r_\pi \gg R_B)$$



- Step 2: For base shot noise:

$$\overline{i_{out}^2}|_{n_{B_{shot}}} = g_m^2 \times (r_\pi^2 \times \overline{i_{n_B}^2}) = (g_m r_\pi)^2 \times \overline{i_{n_B}^2} \times \left(\frac{R_B}{R_B + r_\pi} \right)^2$$

$$\therefore \overline{i_{out}^2}|_{n_{B_{shot}}} \approx (g_m r_\pi)^2 \times \overline{i_{n_B}^2} \times \left(\frac{R_B}{r_\pi} \right)^2 \quad (\because r_\pi \gg R_B)$$

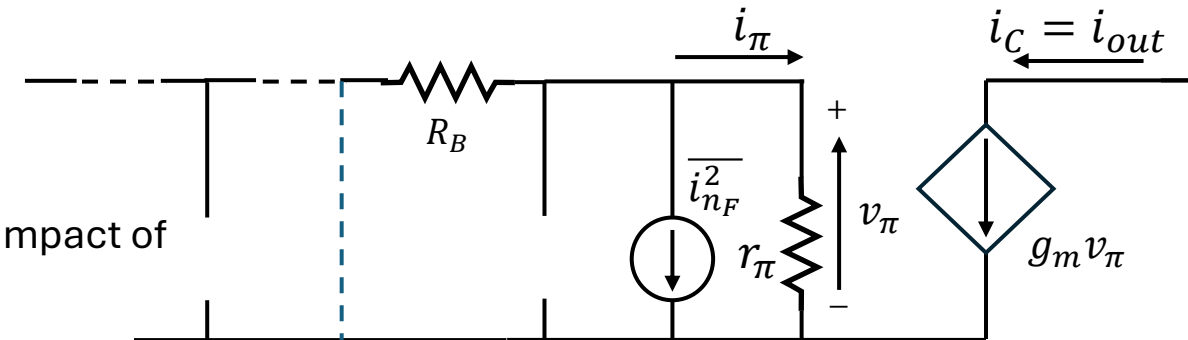


- Step 3: For flicker noise:

- Same calculation as the base shot noise.

$$\therefore \overline{i_{out}^2}|_{n_{Flicker}} \approx (g_m r_\pi)^2 \times \overline{i_{n_F}^2} \times \left(\frac{R_B}{R_B + r_\pi} \right)^2 \quad (\because r_\pi \gg R_B)$$

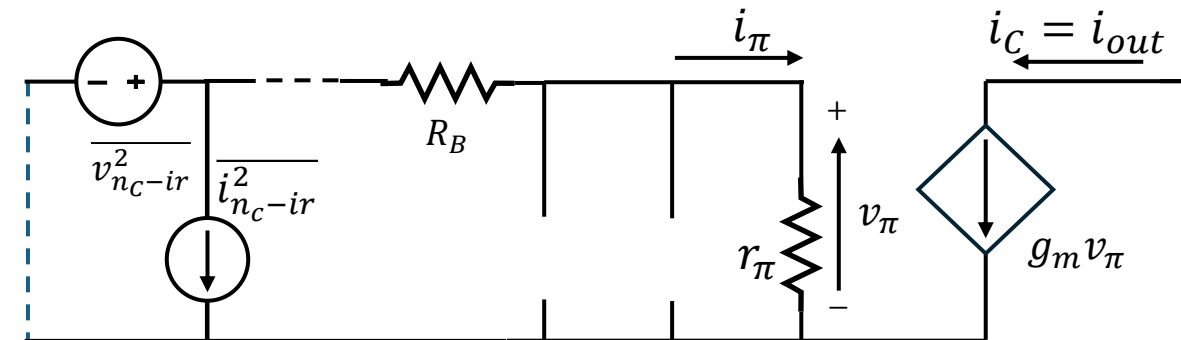
- The presence of the bulk resistance at the base reduces the impact of the flicker and base shot noises at the output.



Modelling Noise: The BJT

Input Referred Noise Model

- For the input referred collector shot noise, we cannot individually transmit them to the output side because they always act together and are correlated.
- Therefore, we calculate the collective voltage they create across r_π in time domain (as opposed to mean squared domain) and then transmit that collective voltage to the output.
- During this calculation, you will see something very interesting:
 - The impact of the correlation between $\overline{v_{n_c-ir}^2}$ and $\overline{i_{n_c-ir}^2}$ is only visible if the input signal source is non-ideal, hence has a source resistance R_S !
 - This will confirm the whole point of the 2-port theory for input referred noise and acts as an example why we need **both** a voltage **and** a current source at the input when we build the input-referred noise model for a two-port device.



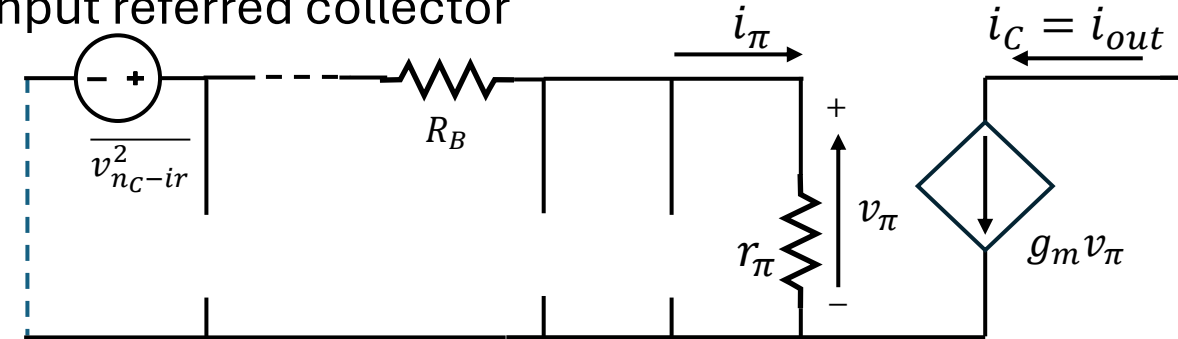
Modelling Noise: The BJT

Input Referred Noise Model

- From superposition principle:

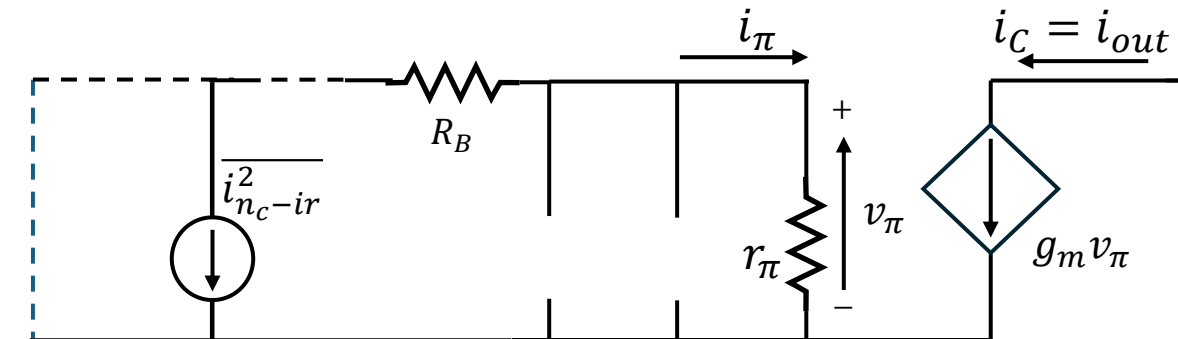
- Step 4: v_π created by the **voltage** component of the input referred collector current shot noise:

- $$v_\pi(t)|_{v_{n_{C-ir}}} = v_{n_{C-ir}}(t) \times \left(\frac{r_\pi}{R_B + r_\pi} \right)$$



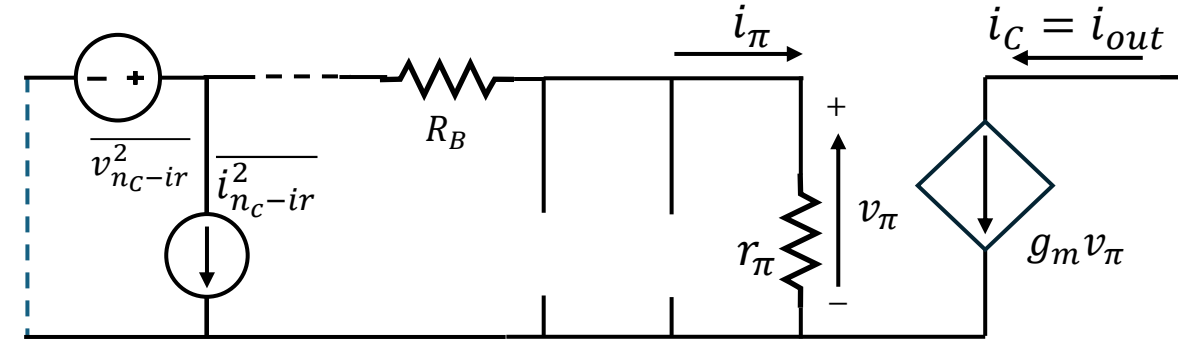
- Step 5: v_π created by the **current** component of the input referred collector current shot noise:

- $$v_\pi(t)|_{i_{n_{C-ir}}} = i_{n_{C-ir}}(t) \times 0$$
 - Since the left leg of R_B is shorted to ground, the current takes this path of least impedance to ground!
 - Therefore, the impact of this current component is only felt in the presence of a source resistance!



Modelling Noise: The BJT Input Referred Noise Model

- $\therefore v_{\pi}(t) = \left(v_{n_{C-ir}}(t) \times \frac{r_{\pi}}{R_B + r_{\pi}} \right) + (i_{n_{C-ir}}(t) \times 0)$
- Now calculate the mean square of the whole:
 - $\therefore \overline{v_{\pi}^2} = \left(\frac{r_{\pi}}{R_B + r_{\pi}} \right)^2 \times \overline{v_{n_{C-ir}}^2}$
 - $\therefore \overline{i_{out}^2}|_{n_{C-ir}} = g_m^2 \times \overline{v_{\pi}^2} = (g_m r_{\pi})^2 \times \frac{\overline{v_{n_{C-ir}}^2}}{(R_B + r_{\pi})^2}$



Modelling Noise: The BJT

Input Referred Noise Model

- Therefore, total output noise current:
 - Since the correlation component of the input referred shot noise was already considered, we can simply add the noise mean square terms together.
- $\overline{i_{out}^2} = \overline{i_{out}^2}|_{n_{B_{Thermal}}} + \overline{i_{out}^2}|_{n_{B_{shot}}} + \overline{i_{out}^2}|_{n_{Flicker}} + \overline{i_{out}^2}|_{n_{C-ir}}$
- $\overline{i_{out}^2} = \left(\frac{g_m r_\pi}{R_B + r_\pi}\right)^2 \times \left(\overline{v_{n_B}^2} + \overline{v_{n_{C-ir}}^2} + \left(\overline{i_{n_B}^2} + \overline{i_{n_F}^2}\right) R_B^2\right)$
- Since R_B is small, the effect of the $\left(\overline{i_{n_B}^2} + \overline{i_{n_F}^2}\right)$ terms vanish, and the $\left(\frac{g_m r_\pi}{R_B + r_\pi}\right)^2 \approx \left(\frac{g_m r_\pi}{r_\pi}\right)^2 = g_m^2$
 - $\therefore \overline{i_{out}^2} \approx g_m^2 \times \left(\overline{v_{n_B}^2} + \overline{v_{n_{C-ir}}^2}\right)$
- It is the base shot noise and the voltage component of the input referred collector current shot noise that dominates!
 - Note however that this is in the presence of an ideal signal source with $R_S = 0$.

Modelling Noise: The BJT

Input Referred Noise Model

- As a homework, repeat the calculation for the total output noise current for BJT connected to a practical signal source with non-zero source impedance ($R_S \neq 0$).
- You will see that for the base thermal noise, base shot noise, and flicker noise, replacing R_B with $R_B + R_S$ is the only required modification to the final output.

- $$\overline{i_{out}^2}|_{n_{B_{Thermal}}} = g_m^2 \times \overline{v_{n_B}^2} \times \left(\frac{r_\pi}{R_B + R_S + r_\pi} \right)^2$$

- $$\overline{i_{out}^2}|_{n_{B_{shot}}} = (g_m r_\pi)^2 \times \overline{i_{n_B}^2} \times \left(\frac{R_B + R_S}{R_B + R_S + r_\pi} \right)^2$$

- $$\overline{i_{out}^2}|_{n_{Flicker}} = (g_m r_\pi)^2 \times \overline{i_{n_F}^2} \times \left(\frac{R_B + R_S}{R_B + R_S + r_\pi} \right)^2$$

- For the **voltage** and **current** components of the input referred collector shot noise the expression will no longer be simple.

- Previously, we had time domain $v_\pi(t) = \left(v_{n_{C-ir}}(t) \times \frac{r_\pi}{R_B + r_\pi} \right) + (i_{n_{C-ir}}(t) \times 0)$ (current noise component vanished), hence mean square $\overline{v_\pi^2} = \left(\frac{r_\pi}{R_B + r_\pi} \right)^2 \times \overline{v_{n_{C-ir}}^2}$

- Let's redo this step for when the source resistance R_S is not zero.

Modelling Noise: The BJT

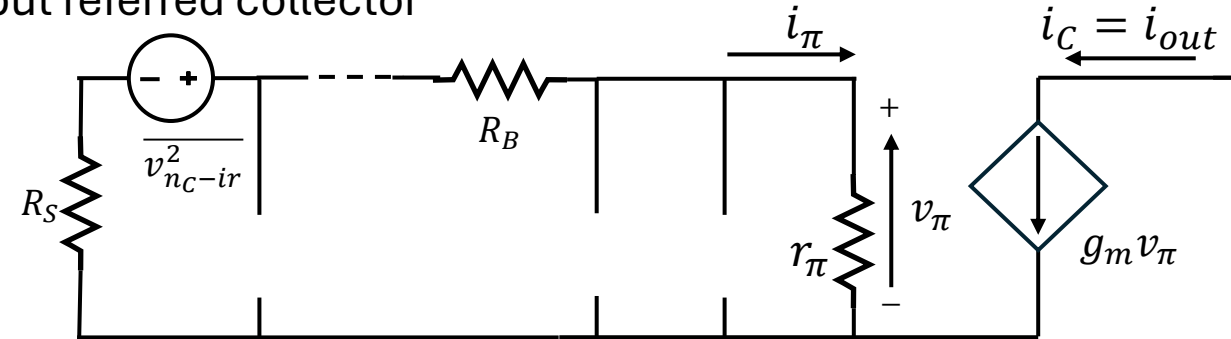
Input Referred Noise Model

- From superposition principle, when source impedance is not zero (non-ideal/practical signal source):

- Step 4': v_π created by the **voltage** component of input referred collector

current shot noise:

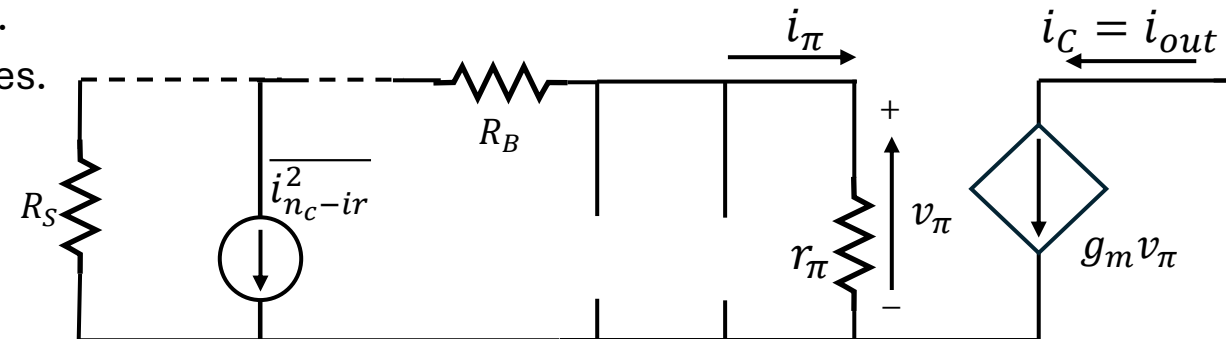
$$v_\pi(t)|_{v_{n_{C-ir}}} = v_{n_{C-ir}}(t) \times \left(\frac{r_\pi}{R_B + R_S + r_\pi} \right)$$



- Step 5': v_π created by the **current** component of the input referred collector current shot noise:

- Now, the left leg of R_B is no longer shorted to ground.
- Therefore, the current divides among the two branches.
- The current divider rule must be applied.

$$v_\pi(t)|_{i_{n_{C-ir}}} = i_{n_{C-ir}}(t) \times \frac{R_S r_\pi}{(R_B + R_S + r_\pi)}$$



Modelling Noise: The BJT

Input Referred Noise Model

- $\therefore v_{\pi}(t) = \left(v_{n_{C-ir}}(t) \times \frac{r_{\pi}}{R_B + R_S + r_{\pi}} \right) + \left(i_{n_{C-ir}}(t) \times \frac{R_S r_{\pi}}{R_B + R_S + r_{\pi}} \right)$
- Now calculate the mean square of the whole:
 - $\therefore \overline{v_{\pi}^2} = \left(\frac{r_{\pi}}{R_B + R_S + r_{\pi}} \right)^2 \times \overline{\left(v_{n_{C-ir}}(t) + R_S \times i_{n_{C-ir}}(t) \right)^2}$
 - $= \left(\frac{r_{\pi}}{R_B + R_S + r_{\pi}} \right)^2 \left(\overline{v_{n_{C-ir}}^2} + R_S^2 \times \overline{i_{n_{C-ir}}^2} + 2R_S \times \overline{v_{n_{C-ir}}(t) \times i_{n_{C-ir}}(t)} \right) \quad (\because \text{See simplifications in lecture 1})$
 - $\therefore \overline{i_{out}^2}|_{n_{C-ir}} = g_m^2 \times \overline{v_{\pi}^2} = \left(\frac{g_m r_{\pi}}{R_B + R_S + r_{\pi}} \right)^2 \times \left(\overline{v_{n_{C-ir}}^2} + R_S^2 \times \overline{i_{n_{C-ir}}^2} + 2 \times R_S \times \overline{v_{n_{C-ir}}(t) \times i_{n_{C-ir}}(t)} \right)$
 - i.e., the cross term does **NOT** vanish!
 - However, since $v_{n_{C-ir}}(t)$ and $i_{n_{C-ir}}(t)$ are **perfectly** correlated, we can write one as the scalar multiple of the other.
 - This math trick simplifies the $\overline{v_{n_{C-ir}}(t) \times i_{n_{C-ir}}(t)}$ to $\sqrt{\overline{v_{n_{C-ir}}^2}} \times \sqrt{\overline{i_{n_{C-ir}}^2}}$ which are both known quantities!
- Therefore,
 - $\therefore \overline{i_{out}^2}|_{n_{C-ir}} = \left(\frac{g_m r_{\pi}}{R_B + R_S + r_{\pi}} \right)^2 \times \left(\overline{v_{n_{C-ir}}^2} + R_S^2 \times \overline{i_{n_{C-ir}}^2} + 2 \times R_S \times \sqrt{\overline{v_{n_{C-ir}}^2}} \times \sqrt{\overline{i_{n_{C-ir}}^2}} \right)$

Modelling Noise: The BJT

Input Referred Noise Model

- Therefore, total output noise current in the presence of a source impedance:
 - Since the correlation component of the input referred shot noise was already considered, we can simply add the noise mean square terms together.

$$\overline{i_{out}^2} = \overline{i_{out}^2}|_{n_{B_{Thermal}}} + \overline{i_{out}^2}|_{n_{B_{shot}}} + \overline{i_{out}^2}|_{n_{Flicker}} + \overline{i_{out}^2}|_{n_{C-ir}}$$

$$\overline{i_{out}^2} = \left(\frac{g_m r_\pi}{R_B + R_S + r_\pi} \right)^2 \times \left(\overline{v_{n_B}^2} + \overline{v_{n_{C-ir}}^2} + \left(\overline{i_{n_B}^2} + \overline{i_{n_F}^2} \right) (R_B + R_S)^2 + R_S^2 \times \overline{i_{n_{C-ir}}^2} + 2 \times R_S \times \sqrt{\overline{v_{n_{C-ir}}^2} \times \overline{i_{n_{C-ir}}^2}} \right)$$

- This expression tells us what noise sources to worry about and what sources to not worry about in a BJT amplifier circuit if we know the source impedance of our input signal source.
 - Remember, the source signal is not always a microphone, or a small signal that you obtain with a sensor.
 - It could very well be the output of a pre-amplification stage, or a buffer circuit, a filter, etc.
- This analysis put us in a position to analyze why we needed **both** a voltage **and** a current source at the input to represent a single collector shot noise current when deriving the input referred noise model.

Modelling Noise: The BJT

Input Referred Noise Calculation

- Why did we need both a current source **and** a voltage source at the input to fold a single output current source to the input?
 - Can't we just divide the output side current with the current gain $|\beta_{AC}|$? - No.
 - The input-referred noise model must work and yield the **same result independent of the source impedance** of an input signal that we connect.
 - Consider the following common emitter transistor circuit operating in the low frequency range, where we **only** consider the collector current shot noise and ignore all other noise sources ($R_B=0$ - no base thermal noise, base shot noise=0, flicker noise=0).
 - For the input referred noise equivalent for the collector shot noise, suppose we **only** considered the **current source** portion $\overline{i_{n_c-ir}^2} = \frac{\overline{i_{n_c}^2}}{|\beta_{AC}|^2}$.
 - If this is still correct, when we propagate **just this current source portion** of the input referred noise to the output side, we should get the original $\overline{i_{n_c}^2}$.



Modelling Noise: The BJT

Input Referred Noise Calculation

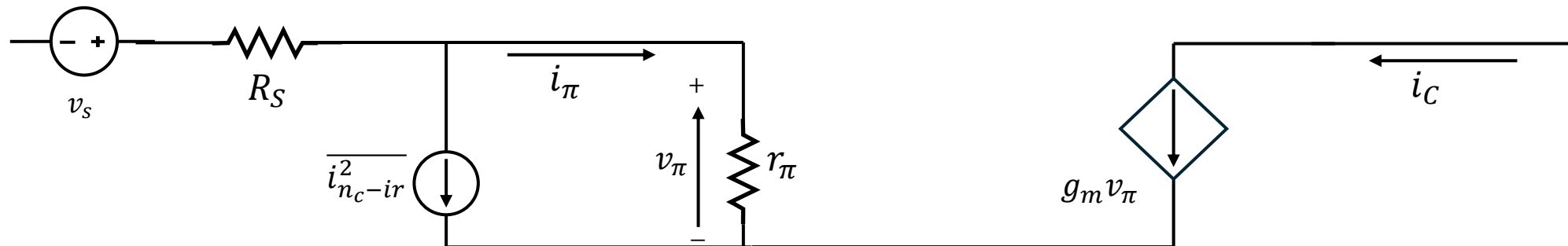
- Output current noise $\overline{i_{n_{output}}^2} = g_m^2 \overline{v_\pi^2} = (g_m r_\pi)^2 \times \overline{i_\pi^2} = (g_m r_\pi)^2 \times \overline{i_{n_c - ir}^2} = (g_m r_\pi)^2 \times \frac{\overline{i_{n_c}^2}}{|\beta_{AC}|^2}$
 - Since we assumed low frequency, C_μ and C_π are zero.
 - $\therefore |\beta_{AC}|^2 = \frac{\beta_{DC}^2}{1 + (\omega r_\pi (C_\pi + C_\mu))^2} = \frac{\beta_{DC}^2}{1 + (0)^2} = \beta_{DC}^2 = (g_m r_\pi)^2$
 - Therefore, $\overline{i_{n_{output}}^2} = \overline{i_{n_c}^2}$
 - This is the expected solution.



Modelling Noise: The BJT

Input Referred Noise Calculation

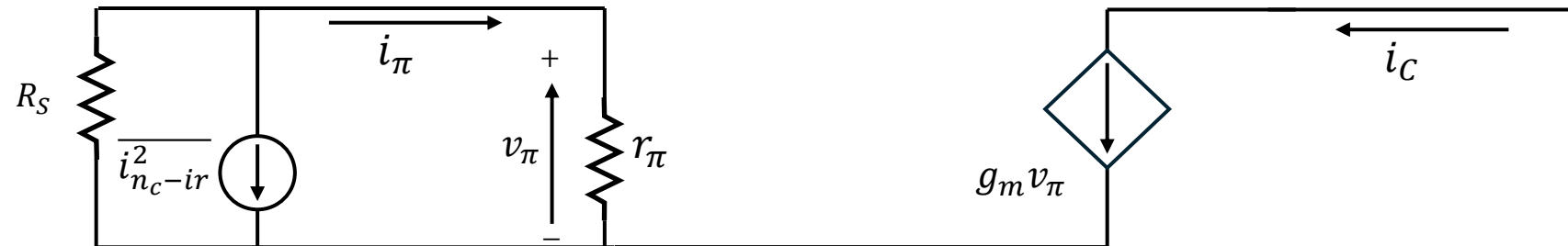
- Then why did we need an additional voltage noise source for the input referred noise model?
 - To understand this, we need to consider a real application where we have a signal source to the input.
 - Let's assume that the source will have an impedance of R_S
 - Let's propagate **just the current noise portion** of the input referred noise to the output side **in the presence of a source impedance**:



Modelling Noise: The BJT

Input Referred Noise Calculation

- Now, the input referred current noise is divided between r_π and R_S .
- From current divider rule we have $\overline{i_\pi^2} = \left(\frac{R_S}{R_S + r_\pi}\right)^2 \times \overline{i_{n_c - ir}^2}$
- Therefore, output current noise $\overline{i_{n_{output}}^2} = g_m^2 \overline{v_\pi^2} = (g_m r_\pi)^2 \times \overline{i_\pi^2} = (g_m r_\pi)^2 \times \left(\frac{R_S}{R_S + r_\pi}\right)^2 \times \overline{i_{n_c - ir}^2}$
 - $\Rightarrow (g_m r_\pi)^2 \times \left(\frac{R_S}{R_S + r_\pi}\right)^2 \times \frac{\overline{i_{n_c}^2}}{|\beta_{AC}|^2}$
 - From low frequency assumption: $\overline{i_{n_{output}}^2} = \left(\frac{R_S}{R_S + r_\pi}\right)^2 \times \overline{i_{n_c}^2}$
 - $\therefore \overline{i_{n_{output}}^2} \neq \overline{i_{n_c}^2}$
 - This is NOT the expected solution.

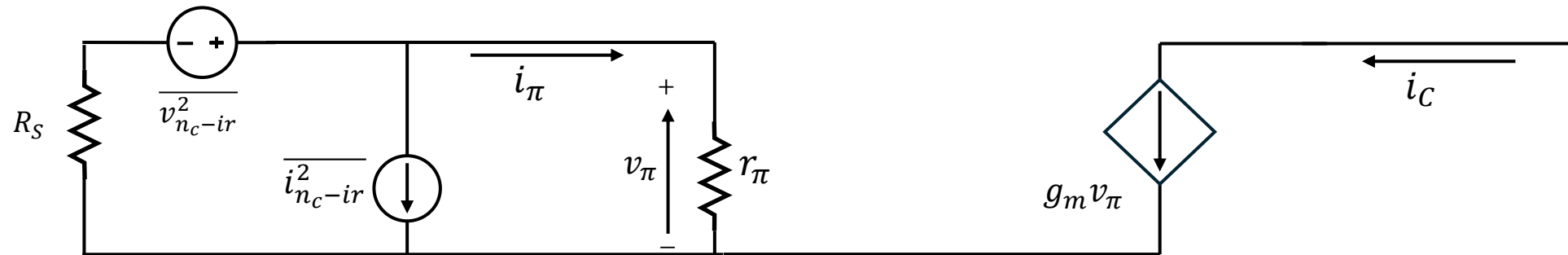


Modelling Noise: The BJT

Input Referred Noise Calculation

- Now let's see what would have happened had we used **both** the current **and** the voltage portions of the input referred collector shot current:

- Here, $\overline{v_{n_{C-ir}}^2} = \frac{\overline{i_{n_C}^2}}{g_m^2}$ and $\overline{i_{n_{C-ir}}^2} = \frac{\overline{i_{n_C}^2}}{|\beta_{AC}|^2}$
- How do we calculate the combined effects of these noise sources?
 - From our previous calculations for the propagation of input referred collector current shot noise to the output side:
 - $$\overline{i_{n_{output}}^2} = \left(\frac{g_m r_\pi}{R_B + R_S + r_\pi} \right)^2 \times \left(\overline{v_{n_{C-ir}}^2} + R_S^2 \times \overline{i_{n_{C-ir}}^2} + 2 \times R_S \times \sqrt{\overline{v_{n_{C-ir}}^2}} \times \sqrt{\overline{i_{n_{C-ir}}^2}} \right)$$
 - Remember, we cannot drop the $v_{n_{C-ir}}(t)$ and $i_{n_{C-ir}}(t)$ terms - They are components of one and the same source.
 - Since we assumed $R_B = 0$:
 - $$\overline{i_{n_{output}}^2} = \left(\frac{g_m r_\pi}{R_S + r_\pi} \right)^2 \times \left(\overline{v_{n_{C-ir}}^2} + R_S^2 \times \overline{i_{n_{C-ir}}^2} + 2 \times R_S \times \sqrt{\overline{v_{n_{C-ir}}^2}} \times \sqrt{\overline{i_{n_{C-ir}}^2}} \right)$$



Modelling Noise: The BJT

Input Referred Noise Calculation

- Plugging in values:

$$\begin{aligned} \overline{i_{n_{output}}^2} &= \left(\frac{g_m r_\pi}{R_S + r_\pi} \right)^2 \times \left(\frac{\overline{i_{n_C}^2}}{g_m^2} + R_S^2 \times \frac{\overline{i_{n_C}^2}}{|\beta_{AC}|^2} + 2 \times R_S \times \sqrt{\frac{\overline{i_{n_C}^2}}{g_m^2}} \times \sqrt{\frac{\overline{i_{n_C}^2}}{|\beta_{AC}|^2}} \right) \\ &= \left(\frac{g_m r_\pi}{R_S + r_\pi} \right)^2 \times \overline{i_{n_C}^2} \times \left(\frac{1}{g_m^2} + R_S^2 \times \frac{1}{|\beta_{AC}|^2} + 2 \times R_S \times \frac{1}{g_m} \times \frac{1}{|\beta_{AC}|} \right) \end{aligned}$$

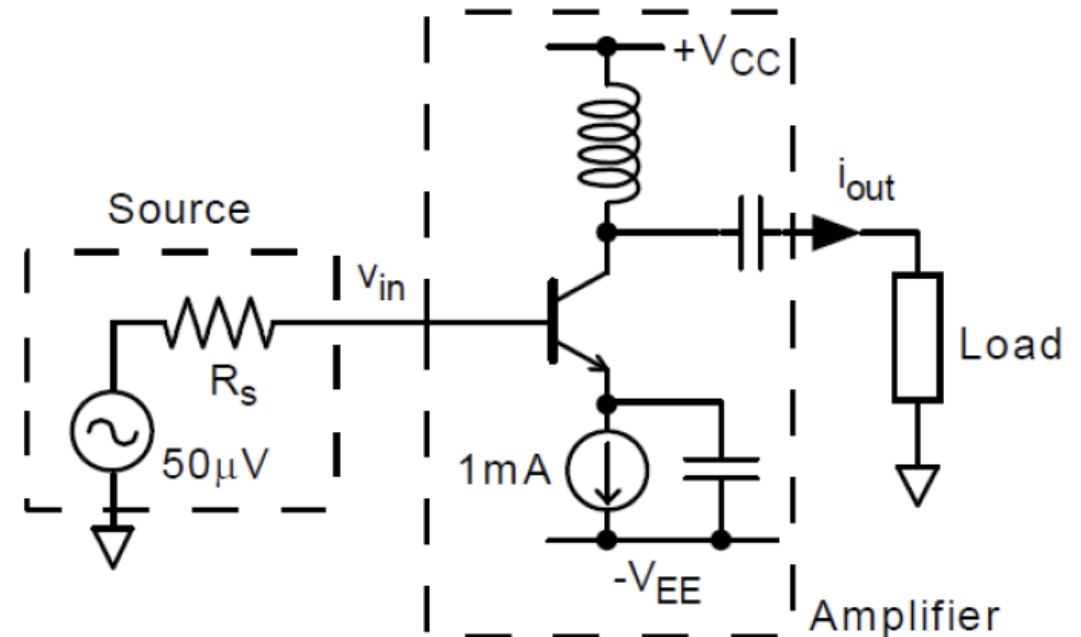
- Since we assumed low frequency operation: $|\beta_{AC}| = \beta_{DC} = (g_m r_\pi)$
 - $= \frac{1}{(R_S + r_\pi)^2} \times \overline{i_{n_C}^2} \times (r_\pi^2 + R_S^2 + 2R_S r_\pi) = \frac{1}{(R_S + r_\pi)^2} \times \overline{i_{n_C}^2} \times (R_S + r_\pi)^2$
- $\therefore \overline{i_{n_{output}}^2} = \overline{i_{n_C}^2}$
 - This **is** the expected solution.

- Therefore, we need **both** the current and voltage noise source portions of the input referred noise model to get the correct behavior in the presence of source impedance.

Modelling Noise: The BJT

Exercise 1

- Suppose the transistor has a current gain of $\beta=50$. Moreover, consider all capacitors to be noiseless and a short circuit to AC signals. Similarly, take the inductor to be noiseless and an open circuit to AC signals. The effective bandwidth of the amplifier is 100 kHz, and in the BJT model use $g_m=38\text{ mS}$ and $r_\pi=1.33\text{ k}\Omega$.
 - Tests performed on this amplifier give that with source impedance, $R_s = 50\ \Omega$ the noise figure is 3dB, and with a source resistance of $R_s = 500\ \Omega$ the noise figure is 9dB.
 - Find the transistor input referred voltage and current noise spectral densities (v_n^2/Hz and i_n^2/Hz) by assuming in both cases that the noise sources are white.



Modelling Noise: The Diode and The BJT

Important Take Homes

- Noise is fundamentally an AC quantity and is a small signal in itself.
 - Begin with small signal models for analysis.
 - Dynamic resistances in small signal models do **NOT** generate noise, because they are not true physical resistances, but represent a behavioral relationship between voltage and current.
- Bulk resistances, terminals, etc. have physical resistance, hence generate thermal noise, and must be considered.
- We mostly ignored the capacitances in diode and BJT small signal models during most noise derivations.
 - Derive/estimate/think about how including these capacitances would change the noise characteristics.
 - Think in terms of the concept of effective bandwidth.
- A diode has thermal noise, shot noise and flicker noise.
 - You need to remember where these noise sources should be placed in a small signal model, and why.
- A BJT has several intrinsic noise sources out of which 4 are important.
 - You need to be able to calculate the AC current gain β_{AC} for a BJT and understand why it is important.
 - Refer noise sources to the input so that a generic noise model can be calculated
 - Understand how the 2-port nature of a BJT required both an equivalent voltage **and** current source at the input for a complete noise model that works independent of the source impedance.

Modelling Noise: The Diode and The BJT

Important Take Homes

- Explain how the source impedance affects the noise flowing through a BJT, and understand how certain noise sources (voltage or current) become more prominent when the source impedance is high than when low.
 - Calculate the output noise through a BJT amplifier circuit.
- Use superposition to calculate the effect of individual sources as they pass through a system.
- Understand how input referred noise sources may not always be independent.
 - In such situations, the calculations become tricky.
- You do NOT need to derive formulae for the exams, but you need to **understand** the concepts used during derivations.
 - For example, you must be able to explain why certain simplification steps are valid in a given derivation.