



University of Moratuwa
Wisdom is all wealth

Noise and Interference in Instrumentation Part 4

Anton F. P. Van Putte, Electronic measurement systems, Taylor & Francis, 2nd Edition
M. M. S. Anand, Electronic instruments and Instrumentation Technology, PHI

The Op-Amp Noise Model

Modelling Noise: Op-Amps

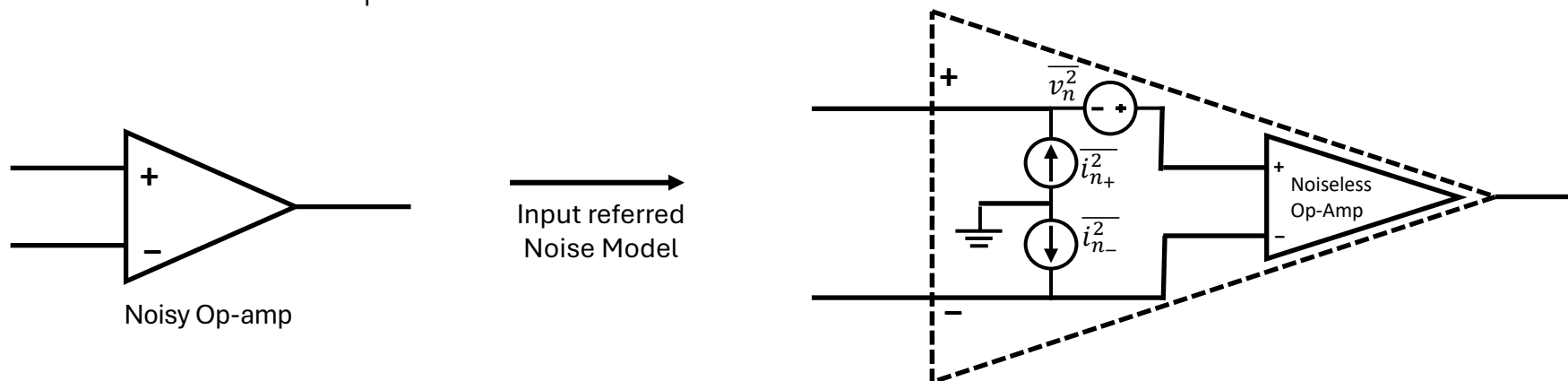
How many ports?

- An op-amp is generally a 5 terminal device (V_+ , V_- , V_{SS} , V_{DD} , V_{OUT}).
 - If we consider all these terminals and the ways signals can be fed into the input terminals, an op-amp would in general be a 5-port device (each terminal forms a port with respect to ground), with noise present at all ports including the power rails.
 - However, for this analysis, let us assume the PSRR is infinite, hence we do not have to worry about noise leaking in through the power rails.
 - This brings the port count down to 3.
 - A complete op-amp amplifier circuit like an inverting amplifier can be considered a 2-port device, because the input is provided only to one port referenced to ground and so is the output.
 - However, there are other applications, ex: the difference amplifier, requires independent inputs at both input terminals.
 - Therefore, in the general case, the op-amp must at least be analyzed as a 3-port device.
 - Therefore, we can no longer represent all the noise sources as a single input voltage noise source and a parallel current noise source across the two input terminals.

Modelling Noise: Op-Amps

The Standard Noise Model

- Therefore, the standard op-amp noise model uses three noise sources:
 - One voltage noise source in series with either of the two input terminals.
 - This represents the **differential** input-referred voltage noise of the op-amp.
 - It doesn't matter to which terminal it is attached, but conventionally it is the non-inverting input.
 - One current noise source entering the inverting input,
 - The effects of this is felt as a voltage noise created when this noise current flows through the source impedance or whatever impedance (R_{S-}) it sees looking out of the terminal at the inverting input.
 - One current noise source entering the non-inverting input.
 - The effects of this is felt as a voltage noise created when this noise current flows through the source impedance or whatever impedance (R_{S+}) it sees looking out of the terminal at the non-inverting input.



Modelling Noise: Op-Amps

The Standard Noise Model

- The reason for having two separate current noise sources at each input terminal is because each can have separate source impedances, thereby changing the amplified noise.
- The reason we only have one input referred voltage noise source:
 - The op-amp essentially works in differential mode.
 - It amplifies the difference ($V_+ - V_-$).
 - Therefore, even if you were to use two separate voltage noise sources at the two input terminals, they will act as a single source when the noise is transferred to the output side.
- Datasheets typically provide only a single input noise current value i_n despite the model having two input current noises i_{n+} and i_{n-} .
 - This is because, by symmetry of the input stages of the underlying differential pairs, i_{n+} and i_{n-} have equal magnitudes (unless the datasheet gives separate values).

Modelling Noise: Op-Amps

The Standard Noise Model

- Where do the noise sources of an op-amp come from?
 - An op-amp is a sophisticated device with a lot of internal components including resistors, capacitors, BJTs/MOSFETs, etc.
 - Each can generate thermal noise, shot noise, flicker noise, etc.
 - Ex: The op-amp is basically a differential pair.
 - If you assume a BJT based op-amp, you will have two independent bias currents going into each of the two input BJTs on either leg.
 - This alone will generate shot noise at the input.
 - Since it is difficult to model all these, we resort to the values for the input-referred current and voltage noise sources provided in the datasheet as measured/calculated by the manufacturer.
 - Remember how many noise sources there were in the BJT, and how the calculations became complex in the presence of source impedances, etc.
 - The input-referred noise sources are usually assumed to be uncorrelated. This is a good approximation for a simple input differential pair, but in op-amps with input bias-current cancellation the two input current noises can be significantly correlated.

Modelling Noise: Op-Amps

The Standard Noise Model

- The input referred voltage noise amplitude spectral density and current noise amplitude spectral density of an op-amp are specified in the datasheet as a curve and/or as typical values at specific frequencies (here, the current noise is given only as a single value).

V_n	Equivalent input noise voltage	$f = 1 \text{ kHz}$	25°C	21	$\text{nV}/\sqrt{\text{Hz}}$
		$f = 10 \text{ kHz}$	25°C	17	
$V_{N(\text{PP})}$	Peak-to-peak equivalent input noise voltage	$f = 0.1 \text{ Hz to } 1 \text{ Hz}$	25°C	0.33	μV
		$f = 0.1 \text{ Hz to } 10 \text{ Hz}$		0.86	
I_n	Equivalent input noise current	$f = 100 \text{ Hz}$	25°C	0.6	$\text{fA}/\sqrt{\text{Hz}}$

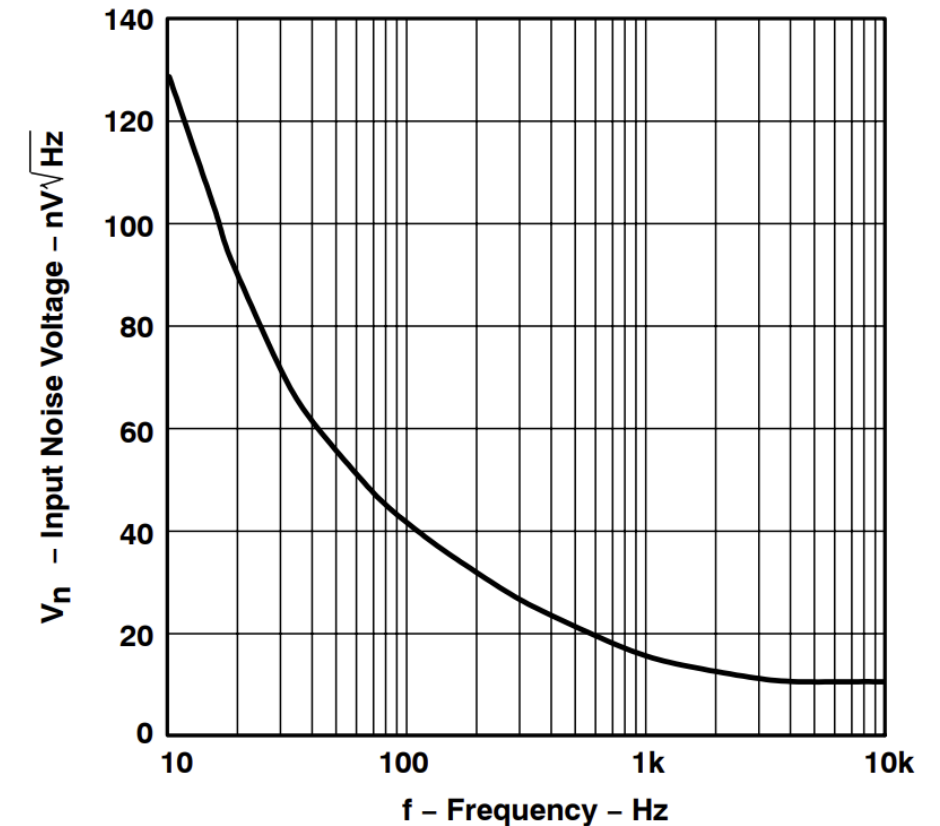
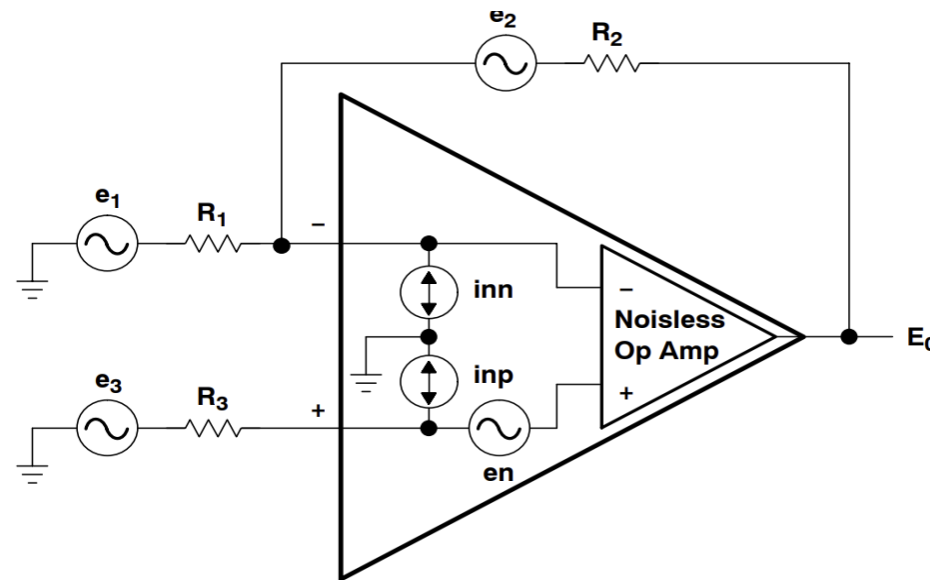


Figure and table from the datasheet of TLV2772

Modelling Noise: Op-Amps

Op-Amp Noise calculations

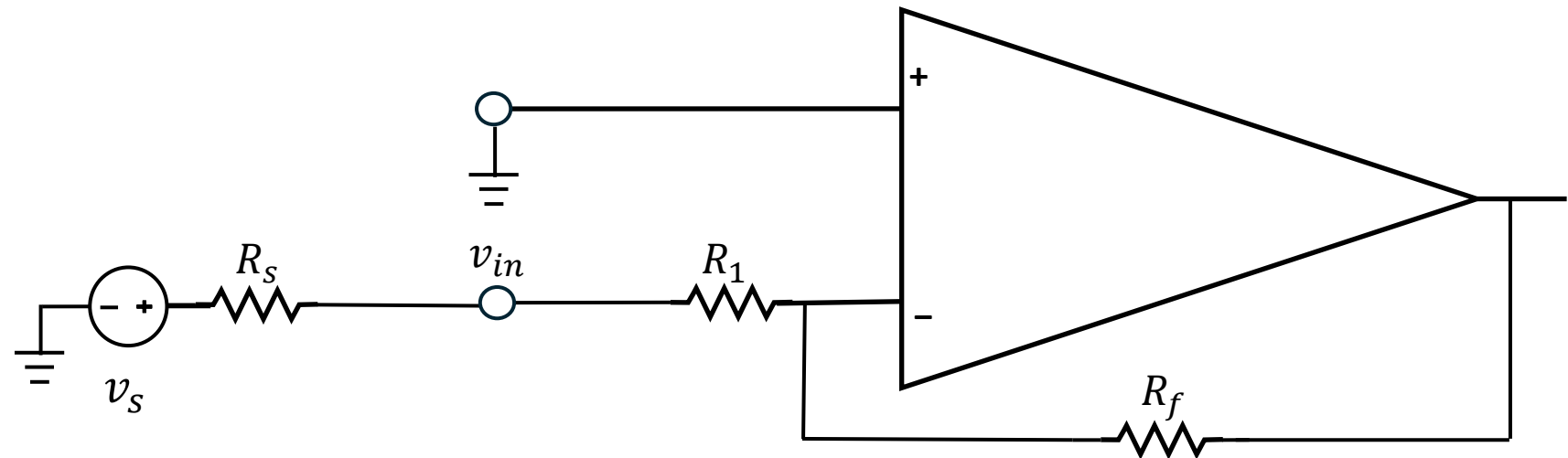
- In addition to these intrinsic noise sources, it is also important to consider noise introduced via the external biasing/gain resistors typically used in an op-amp circuit.
 - These will generate their own thermal noise as well as dependent voltage noise due to the current noise terms passing through them.
 - The following article guides through the calculation of noise output in an op-amp circuit:
 - <https://www.ti.com/lit/an/slva043b/slva043b.pdf>



Example Calculation

Total Output Noise Calculation

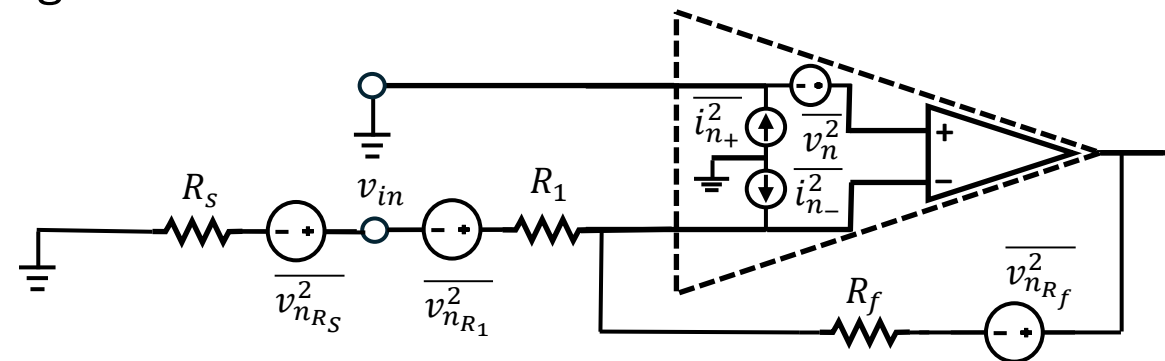
- Consider the following simple example:
 - We have an inverting op-amp configuration as follows.
 - The signal source has an impedance of R_S .
 - Derive an expression for the total output noise from this simple inverting amplifier circuit.



Example Calculation

Contribution from External Resistors to Output Noise

- Replace the op-amp with its input referred noise model and add thermal noise to each resistor.
 - Note: I have grounded the input signal voltage as it has no implications on the noise calculations.
 - This is allowed by the superposition principle for sources.
 - Any source that is on the inverting input gets amplified by the inverting gain $G_I = -\frac{R_f}{R_-}$ where R_- is the impedance between the inverting input and signal ground, excluding R_f .
 - In the presence of a source impedance, $R_- = R_S + R_1 \Rightarrow G_I = -\frac{R_f}{R_S + R_1}$
 - Similarly, any source on the non-inverting input gets amplified by the non-inverting gain $G_N = 1 + \frac{R_f}{R_-}$.
 - In the presence of a source impedance, $R_- = R_S + R_1 \Rightarrow G_N = 1 + \frac{R_f}{R_S + R_1}$
 - Therefore, understand that the effective gain can change significantly if the source impedance is large!



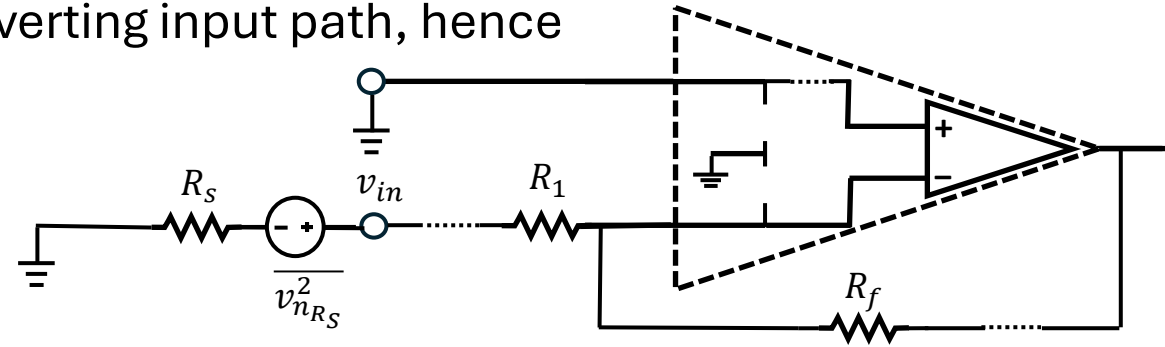
Example Calculation

Contribution from External Resistors to Output Noise

- First, begin with transferring the noise generated by external components connected to the op-amp.

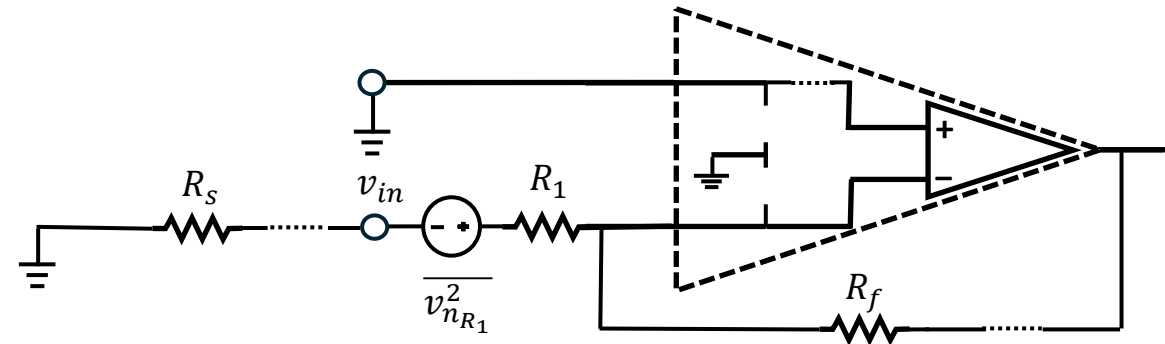
- The thermal noise source $\overline{v_{n_{R_S}}^2}$ is in series with the inverting input path, hence amplified by G_I .

$$\overline{v_{n_{out_{ext}}}^2}|_{v_{n_{R_S}}} = G_I^2 \times \overline{v_{n_{R_S}}^2} = \left(\frac{R_f}{R_S + R_1} \right)^2 \overline{v_{n_{R_S}}^2}$$



- The thermal noise source $\overline{v_{n_{R_1}}^2}$ is also in series with the inverting input path, hence amplified by G_I .

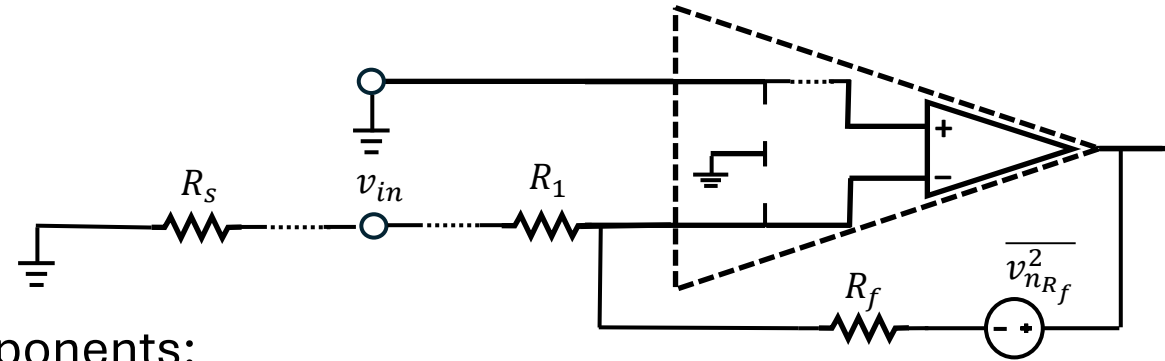
$$\overline{v_{n_{out_{ext}}}^2}|_{v_{n_{R_1}}} = G_I^2 \times \overline{v_{n_{R_1}}^2} = \left(\frac{R_f}{R_S + R_1} \right)^2 \overline{v_{n_{R_1}}^2}$$



Example Calculation

Contribution from External Resistors to Output Noise

- The thermal noise source $\overline{v_{n_{R_f}}^2}$ is directly felt at the output.
 - $\therefore$ amplification is not needed.
 - $\overline{v_{n_{out_{ext}}}^2} |_{v_{n_{R_f}}} = \overline{v_{n_{R_f}}^2}$



- $\therefore$ Total voltage noise at the output from external components:
 - $\overline{v_{n_{out_{ext}}}^2} = G_I^2 \times (\overline{v_{n_{R_S}}^2} + \overline{v_{n_{R_1}}^2}) + \overline{v_{n_{R_f}}^2}$
 - $\Rightarrow \overline{v_{n_{out_{ext}}}^2} = \left(\frac{R_f}{R_S + R_1} \right)^2 \times (\overline{v_{n_{R_S}}^2} + \overline{v_{n_{R_1}}^2}) + \overline{v_{n_{R_f}}^2}$

Example Calculation

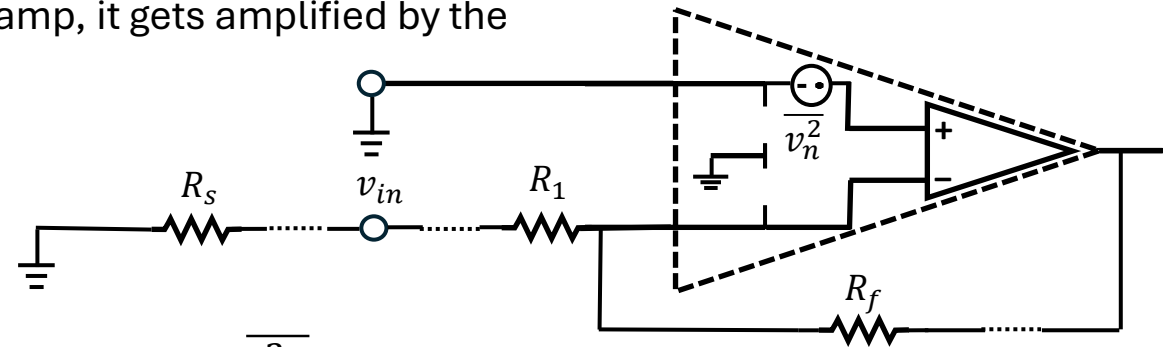
Contribution from Intrinsic Noise Sources to Output Noise

- Now, we transfer the intrinsic noise of the op-amp to the output.

- Intrinsic input referred differential voltage noise $\overline{v_n^2}$:

- Since it is connected to the non-inverting terminal of the op-amp, it gets amplified by the non-inverting gain.

$$\therefore \overline{v_{n_{out_{int}}}^2} |_{v_n} = G_N^2 \times \overline{v_n^2} = \left(1 + \frac{R_f}{R_S + R_1}\right)^2 \times \overline{v_n^2}$$



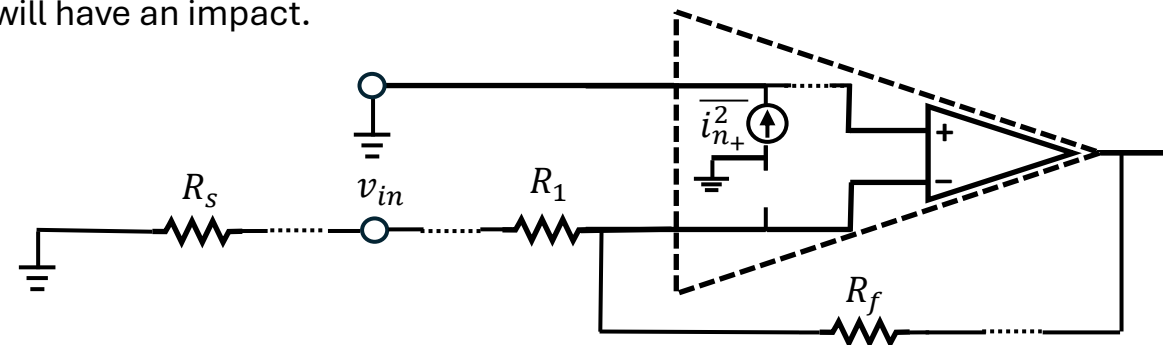
- Intrinsic input referred current noise source on the **non**-inverting input $\overline{i_{n+}^2}$:

- Since there is no resistance on the non-inverting leg, the intrinsic current noise on that leg does **NOT** propagate to the output.

- Therefore, there is no impact from $\overline{i_{n+}^2}$ on the output.

- Note: If there was a resistance on the non-inverting input, $\overline{i_{n+}^2}$ will have an impact.

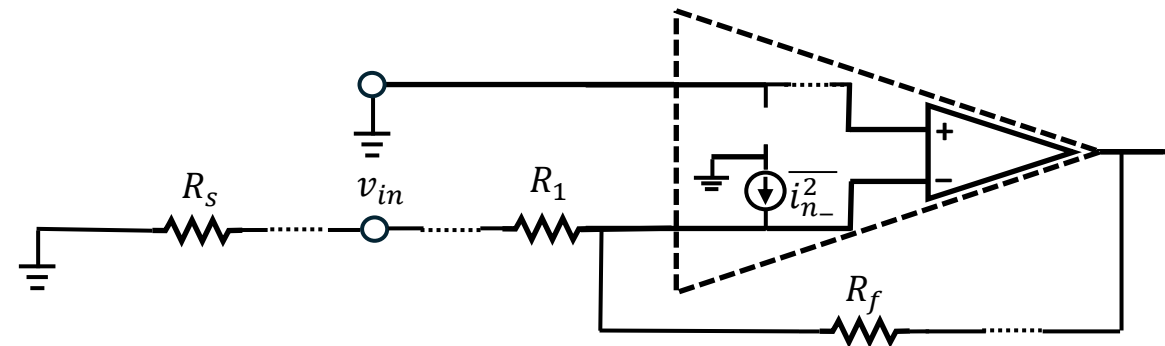
$$\therefore \overline{v_{n_{out_{int}}}^2} |_{i_{n+}} = 0$$



Example Calculation

Contribution from Intrinsic Noise Sources to Output Noise

- Intrinsic input referred current noise source on the **inverting** input $\overline{i_{n-}^2}$:
 - This current noise has two paths to flow through: Through $(R_1 + R_s)$ and through R_f .
 - However, the negative feedback ensures that the inverting and non-inverting inputs are a virtual short ($V_+ - V_- = 0$).
 - Since the non-inverting input is at ground (after shorting the intrinsic differential voltage noise source via superposition principle), both ends of the $(R_1 + R_s)$ resistor pair are at ground.
 - Therefore, no current flows through it.
 - Therefore, all the current from $\overline{i_{n-}^2}$ flows through R_f
 - $\therefore \overline{v_{n_{out_{int}}}^2}|_{i_{n-}} = \overline{i_{n-}^2} \times R_f^2$
- Therefore, overall effect of intrinsic noise sources on the output voltage noise:
 - $\overline{v_{n_{out_{int}}}^2} = G_N^2 \times \overline{v_n^2} + \overline{i_{n-}^2} \times R_f^2$
 - $\Rightarrow \overline{v_{n_{out_{int}}}^2} = \left(\left(1 + \frac{R_f}{R_s + R_1} \right)^2 \times \overline{v_n^2} \right) + \left(R_f^2 \times \overline{i_{n-}^2} \right)$



Example Calculation

Total Output Noise

- Now, we add all output referred noise sources to get the total output noise voltage.

- $$\overline{v_{n_{out}}^2} = \overline{v_{n_{out_{ext}}}^2} + \overline{v_{n_{out_{Int}}}^2}$$

- $$\Rightarrow \overline{v_{n_{out}}^2} = \left\{ \left(\frac{R_f}{R_S + R_1} \right)^2 \times (\overline{v_{n_{R_S}}^2} + \overline{v_{n_{R_1}}^2}) \right\} + \overline{v_{n_{R_f}}^2} + \left\{ \left(1 + \frac{R_f}{R_S + R_1} \right)^2 \times \overline{v_n^2} \right\} + (R_f^2 \times \overline{i_{n_-}^2})$$

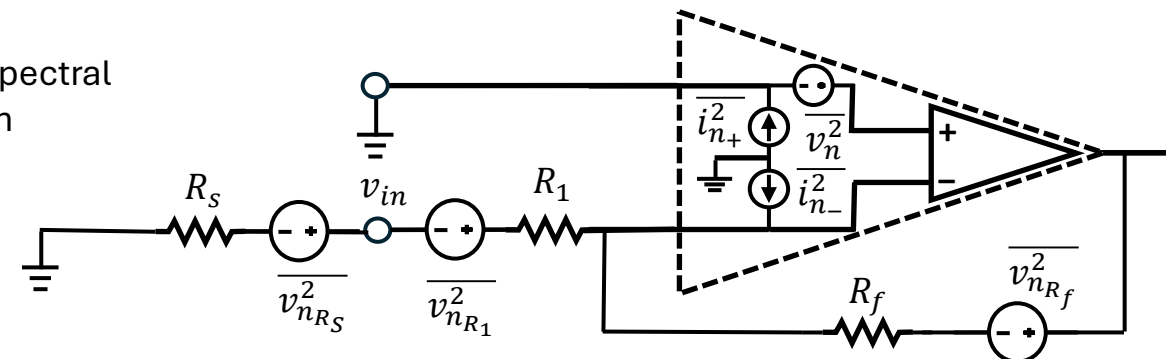
- Substituting the thermal noise terms and assuming equal temperature on all resistors, we have:

- $$\therefore \overline{v_{n_{out}}^2} / \Delta f = 4k_B T \left(\left(\frac{R_f}{R_S + R_1} \right)^2 (R_S + R_1) + R_f \right) + \left\{ \left(1 + \frac{R_f}{R_S + R_1} \right)^2 \times \overline{v_n^2} \right\} + (R_f^2 \times \overline{i_{n_-}^2})$$

- $$= 4k_B T \left(\frac{R_f^2}{R_S + R_1} + R_f \right) + \left\{ \left(1 + \frac{R_f}{R_S + R_1} \right)^2 \times \overline{v_n^2} \right\} + (R_f^2 \times \overline{i_{n_-}^2})$$

- $$\Rightarrow \overline{v_{n_{out}}^2} / \Delta f = 4k_B T R_f G_N + G_N^2 \overline{v_n^2} + R_f^2 \overline{i_{n_-}^2}$$

- $\overline{v_n^2}$ and $\overline{i_{n_-}^2}$ are given in the datasheet of an op-amp as spectral densities (V^2/Hz and A^2/Hz), which is how they are used in the $/\Delta f$ expressions above.



General Noise Considerations: Component Selection

The Impact of Source Impedance

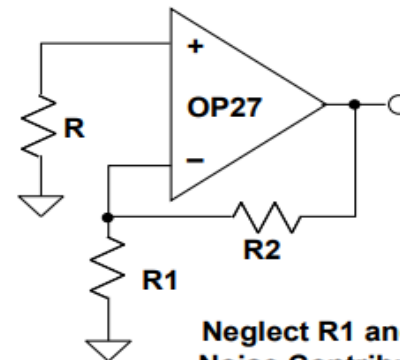
- During the calculation for the contribution from the op-amp's intrinsic noise to the total output noise of an inverting amplifier, we found that:
 - $\overline{v_{n_{out_{int}}}^2} = (G_N^2 \times \overline{v_n^2}) + (R_f^2 \times \overline{i_{n-}^2})$
 - We can refer this quantity back to the input by dividing by the inverting gain (because this is an inverting amplifier circuit) :
 - $\overline{v_{n_{out_{int}}-ir}^2} = \frac{\overline{v_{n_{out_{int}}}^2}}{G_I^2} = \frac{(\overline{v_n^2} \times G_N^2)}{G_I^2} + \frac{(R_f^2 \times \overline{i_{n-}^2})}{G_I^2} = \overline{v_n^2} \left(\frac{R_S + R_1 + R_f}{R_f} \right)^2 + \overline{i_{n-}^2} (R_S + R_1)^2$
 - This quantity becomes bigger as R_S increases.
 - Therefore, the SNR drops as R_S increases.
- Therefore, using a low noise device does **not** always guarantee low noise (high SNR) in every application.
 - While the input referred noise voltages and currents are independent of the source, how these get transferred to the output is directly tied with the source impedance.
 - If the op-amp boasts low current noise, this would only be good in a case where the impedance of the signal source is high (because low current going through high resistance will still give a relatively low noise voltage).
 - If on the other hand your signal source already has low impedance, using a low current noise op amp may not be useful.
 - Instead, it may be the thermal noise of the external resistors, or the op-amp's intrinsic voltage noise that now dominate.

General Noise Considerations: Component Selection

The Impact of Source Impedance on Dominant Noise Source

- The following example shows how different sources in an Op-amp circuit's noise dominate with the change in source resistance R .
 - At low R , the intrinsic current noise is downscaled, hence amplifier voltage noise dominates.
 - At intermediate R , the Johnson noise of R itself dominates; the amplifier is already good enough.
 - At high R , the intrinsic current noise flowing through R is scaled up at a rate proportional to R .
 - At the same time. the thermal noise voltage of R increases at a rate proportional to $\sqrt{R}$.

EXAMPLE: OP27
Voltage Noise = $3\text{nV} / \sqrt{\text{Hz}}$
Current Noise = $1\text{pA} / \sqrt{\text{Hz}}$
 $T = 25^\circ\text{C}$



CONTRIBUTION FROM	VALUES OF R		
	0	3k Ω	300k Ω
AMPLIFIER VOLTAGE NOISE	3	3	3
AMPLIFIER CURRENT NOISE FLOWING IN R	0	3	300
JOHNSON NOISE OF R	0	7	70

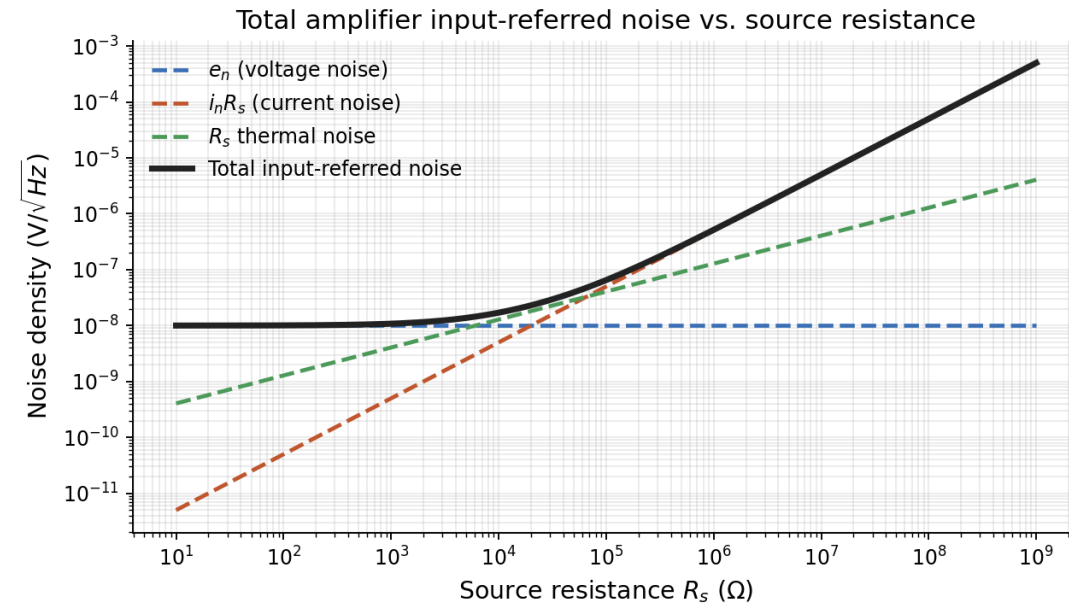
RTI NOISE ($\text{nV} / \sqrt{\text{Hz}}$)
Dominant Noise Source is Highlighted

Source: Analog Devices

General Noise Considerations: Component Selection

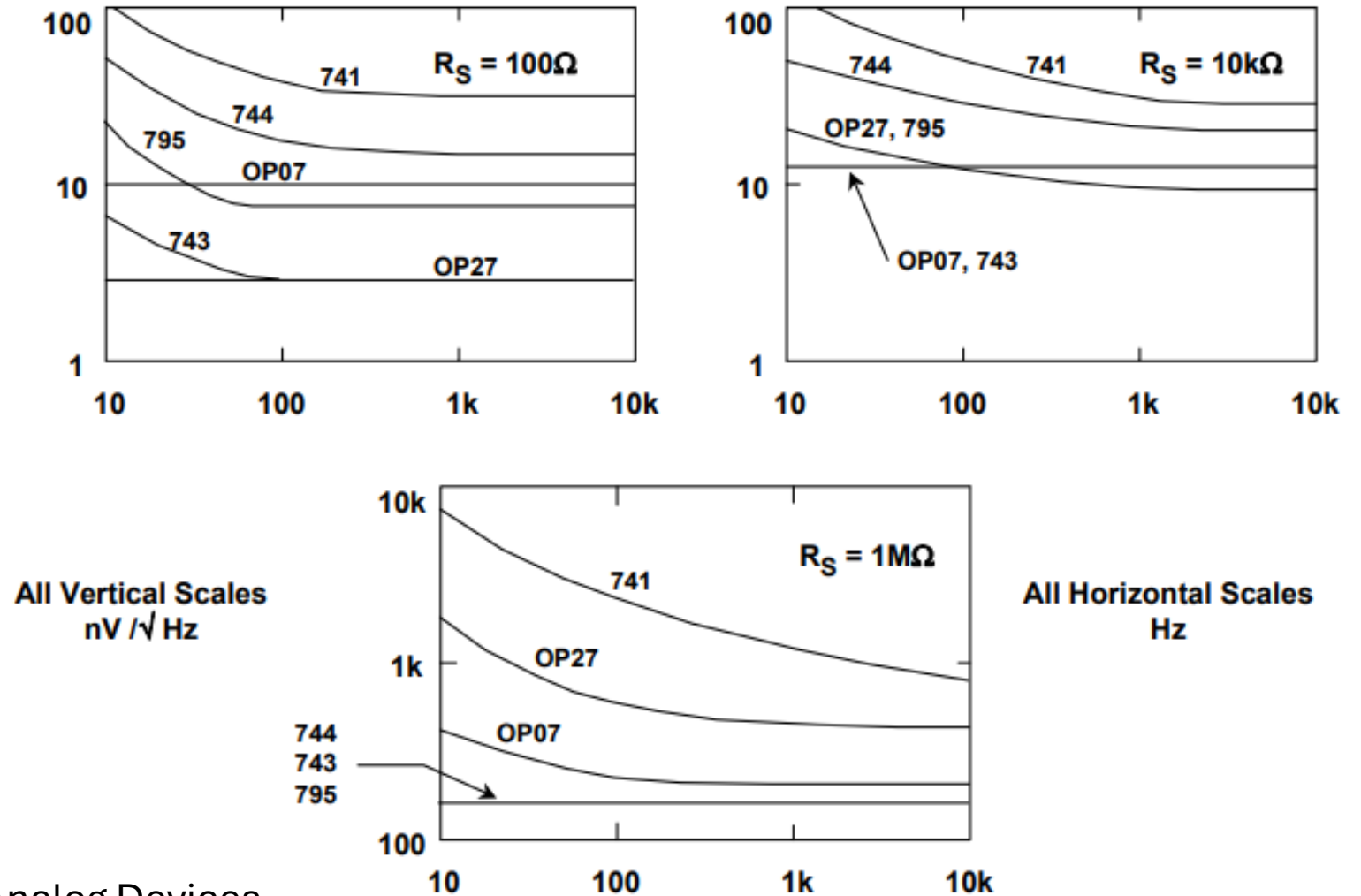
Total Op-Amp Noise vs Source Resistance

- Three regimes are visible when the total op-amp noise is plotted against the source impedance.
 - For low R_s : intrinsic voltage noise (e_n^2) dominates
 - Pick a low- e_n (voltage-noise-optimized) amplifier.
 - For intermediate R_s , $\frac{e_n^2}{4k_B T} < R_s < \frac{4k_B T}{i_n^2}$: R_s 's own thermal noise dominates.
 - The amplifier is already good enough; no amplifier choice can remove this noise floor.
 - For high R_s : intrinsic current noise scaled by R_s becomes dominant when $R_s > \frac{4k_B T}{i_n^2}$, because $i_n R_s$ grows $\propto R_s$ while thermal noise grows only $\propto \sqrt{R_s}$.
 - Pick a low- i_n amplifier (typically MOSFET/JFET input), pushing this crossover to very high R_s .



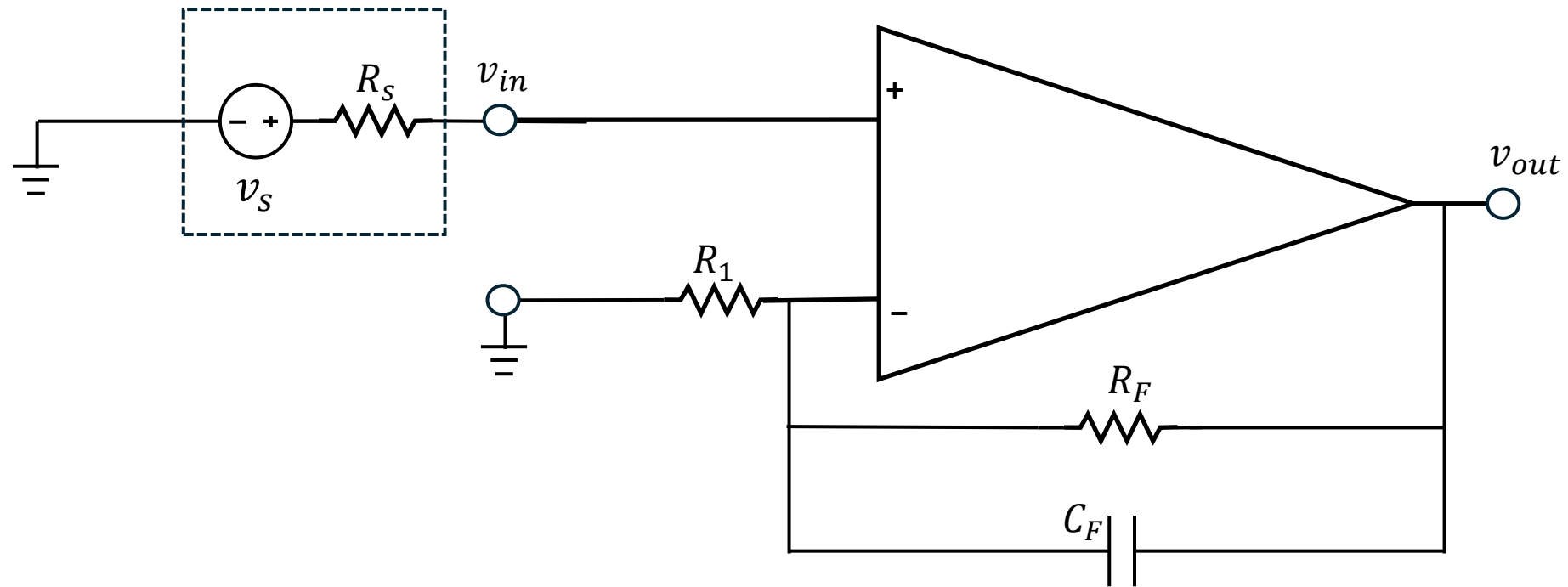
General Noise Considerations: Component Selection

Different Amplifiers Are Best At Different Source Impedances



Source: Analog Devices

Homework



Homework

- A sensor is modelled as an ideal voltage source v_s in series with its source resistance R_s . It drives the non-inverting amplifier shown. Assume the following values: signal $V_{s_{RMS}} = 25 \mu V$, source resistance $R_s = 1 k\Omega$, gain resistor $R_1 = 1 k\Omega$, feedback resistor $R_F = 10 k\Omega$, feedback capacitor $C_F = 1 nF$. Assume that the intrinsic op-amp voltage and current noise from the datasheet are $e_n = 10 nV/\sqrt{Hz}$ and $e_n = 0.1 pA/\sqrt{Hz}$ respectively (assume these are white noise). Assume capacitors are noiseless. $T = 300 K$ and $k_B = 1.38 \times 10^{-23} J/K$. Assume all noise sources are uncorrelated. Assume all noise is band-limited to B_{eff} from (d).
 - a) Find the transfer function of the circuit.
 - b) What is the purpose of this circuit?
 - c) Find the low frequency/DC gain and the corner frequency f_c .
 - d) Calculate the effective bandwidth B_{eff} for noise.
 - e) Calculate the rms thermal noise of the source resistance over B_{eff} . Find the SNR of the source in dB. This is the best SNR the system could ever achieve.
 - f) Draw the amplifier with every noise source included. Convert all noise spectral densities to RMS noise values.
 - g) Refer all noise sources to the non-inverting input (Hint: multiply each source by its own gain to get its output contribution, then divide by the signal gain to get its contribution at the input).
 - h) Find the total input referred noise and the total output noise.
 - i) Find the SNR at the output in dB and find the noise figure.
 - j) Which noise source dominates?
 - k) Now replace the sensor (signal source) with one that has $R_s = 100 k\Omega$ resistance (ex: a dry ECG electrode). Repeat (e), (h) and (i). Does the noise figure get better or worse? Does the output SNR get better or worse? Explain why these two answers point in different directions.

Important Take-Homes

- Op-amps are multi port devices with multiple linear and non-linear components within them.
 - We simplify the noise model to have 2 input current noise sources and 1 input voltage noise source.
- The external resistors we add to an op-amp will add noise on top of the intrinsic noise the op-amp generates.
- When choosing a low noise amplifier, the source impedance should also be considered:
 - Choosing an amplifier with low voltage noise will not help in a situation where the source impedance is high.
 - Choosing an amplifier with low current noise is good in this case.
- Despite choosing a low noise amplifier, the source's own thermal noise (spectral density $4k_BTR_S$) and thermal noise of external components will still set a noise floor that cannot be removed by swapping amplifiers.
- The source impedance can also modify the **gain** of an amplifier.